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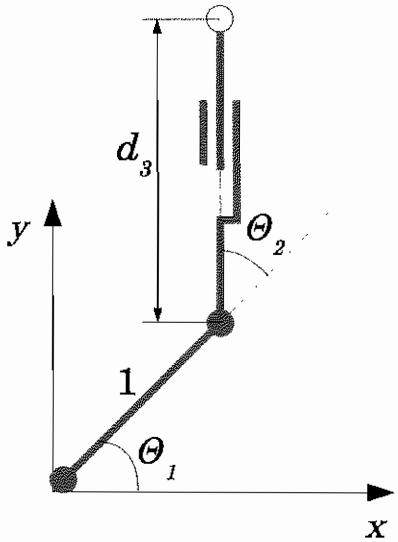
ROBOTICS
Doctoral Qualifying Examination, May 2016
Mechanical Engineering Department, Columbia University

1 Forward and Inverse Kinematics

Consider the planar robot depicted below:



The robot has 2 revolute joints with values Θ_1 and Θ_2 respectively, and one prismatic joint with value d_3. The revolute joints allow continuous rotation without limits, but the prismatic joint has limits $d_3 \in [1, 2]$.

Assume we require the end-effector to be at position $[a, b]^T$, and we do not care about end-effector orientation.

a) Show how to compute values for all the robot joints such that the end-effector achieves the desired position.

b) How many solutions are there in the general case? If the number of solutions depends on certain characteristics of a and b, explain how and why.

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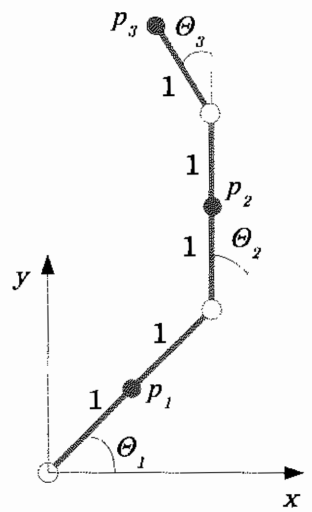
2 Differential Kinematics and Force Generation

Consider the planar robot depicted below:

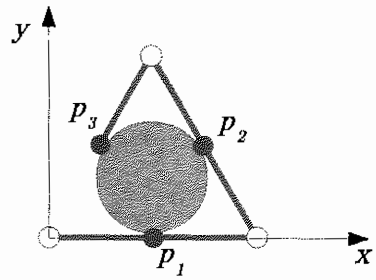


The robot has 3 links and 3 revolute joints. The first two links have length 2; the final link has length 1. We define p_1 as the midpoint of link 1, p_2 as the midpoint of link 2, and p_3 as the end of link 3.

a) Compute the Jacobian matrix $\boldsymbol{J}$ such that $\boldsymbol{J}\dot{\boldsymbol{q}} = \boldsymbol{v}$, where $\dot{\boldsymbol{q}} = [\dot{q_1}, \dot{q_2}, \dot{q_3}]^T$ is the vector of joint velocities and $\boldsymbol{v} = [\dot{p_{1x}}, \dot{p_{1y}}, \dot{p_{2x}}, \dot{p_{2y}}, \dot{p_{3x}}, \dot{p_{3y}}]^T$ contains the Cartesian velocities of points p_1, p_2, p_3.

b)Assuming a non-singular robot configuration, what is $\texttt{rank}(\boldsymbol{J})$? What is $\texttt{dim}(\texttt{im}(\boldsymbol{J}))$, the dimensionality of the image (also known as range) of $\boldsymbol{J}$? What is $\texttt{dim}(\texttt{null}(\boldsymbol{J}))$, the dimensionality of the nullspace of $\boldsymbol{J}$?

Now consider the situation depicted in the image below, where the robot is executing a whole-arm grasp of a circular object. In this case, the robot is making contact at points p_1, p_2 and p_3 defined above, $\Theta_1 = 0$ and $\Theta_2 = \Theta_3 = 120°$.



c) Assuming that the robot and object are frictionless (and thus any contact forces must be normal to the object surface and perpendicular to the link making contact), give an example of joint torques that produce non-zero contact forces such that the object is in equilibrium.

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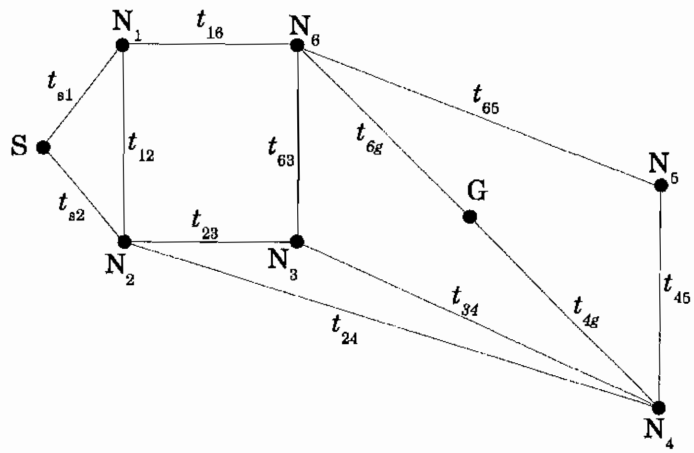
3 Motion Planning

Consider the graph shown below, depicting paths that a mobile robot can take between various nodes. The node that the robot starts in is labeled $\boldsymbol{S}$; the goal node that the robot must reach is labeled $\boldsymbol{G}$.



The variable above each edge indicates the time needed to traverse it, also referred to as the "cost" of traversing the edge. Note that the time to traverse an edge does not have to be proportional to its geometric length. The time to traverse an edge is the same regardless of direction ($e.g. t_{23} = t_{32}$). The purpose of a motion planning algorithm in this case is to find the path to the goal that takes the shortest amount of time (or has the "smallest cost") to traverse.

a) Assign values to the costs of all edges such that **Dijkstra's algorithm visits the goal node last**. For the costs the you have assigned, write the sequence in which Dijkstra's algorithm visits the nodes in the graph. You can write the assigned costs directly next to the edges in the figure above, or in a separate table.

b) Using the same edge cost values you have assigned above, for each node n, assign values to a heuristic function $h(n)$ that can be used by the A* algorithm. The heuristic function values you have assigned must be **acceptable**, and, when using them, **the A* algorithm must find the optimal path by visiting fewer nodes than Dijkstra's algorithm**. Write the values of the heuristic function you have assigned in the table below, and also write the sequence in which the A* algorithm visits the nodes in the graph.

Node n	N1	N2	N3	N4	N5	N6	G
$h(n)$							

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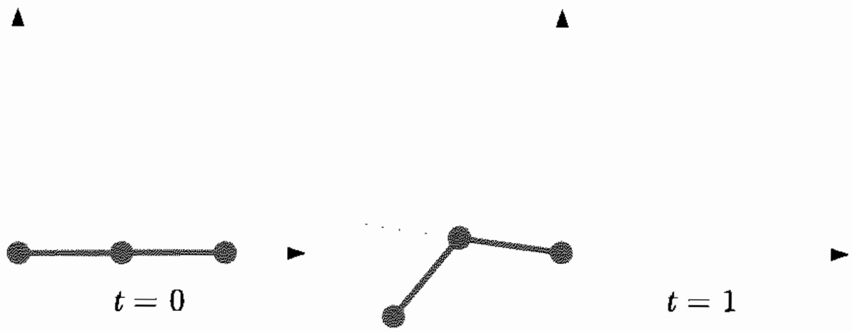
4 Trajectory Computation and Dynamics

A planar 2-link manipulator must execute the trajectory segment shown in the image below:



At $t = 0$, the robot is at rest with joint values $\boldsymbol{q} = [0, 0]^T$. At $t = 1$ the robot must be at rest with joint values $\boldsymbol{q} = [3, 1]^T$. The joints must be synchronized at the start and end of the trajectory (i.e. start and stop moving at the same time), but there are no other synchronization requirements during the trajectory.

a) For each joint q_i with $i \in \{1, 2\}$, compute the coefficients of a cubic polynomial for expressing $q_i(t)$ in order to achieve the conditions above.

b) What is the maximum joint velocity achived by either joint during execution of this trajectory, and at what moment in time does it occur?

c) At the beginning of the trajectory ($t = 0$) what are the joint torques needed to achieve the desired joint accelerations? Assume the following dynamic properties:

- inertia matrix $\boldsymbol{M} = \begin{bmatrix} 6 & 2 \\ 2 & 1 \end{bmatrix}$
- Coriolis and centripetal matrix $\boldsymbol{C} = \begin{bmatrix} 12 & 4.5 \\ 2.5 & 14 \end{bmatrix}$
- gravity vectory $\boldsymbol{G} = [4, 1]^T$
- joints are considered frictionless
- no other external force is applied to the end-effector

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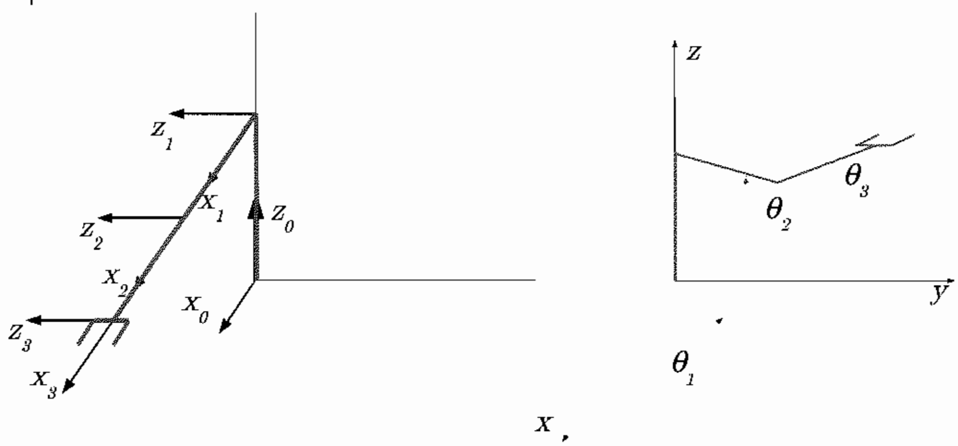
1 Forward and Inverse Kinematics

Assume the following DH configuration for a robot, illustrated below:

i	θ	d	a	α
1	q_1	1	0	90°
2	q_2	0	1	0
3	q_3	0	1	0



Note that, in the used convention, parameters θ_i, d_i, a_i and α_i define the transform from coordinate frame $\{i-1\}$ to coordinate frame $\{i\}$, as illustrated in the picture. These paremeters represent, in order, rotation around the z axis, translation along the z axis, translation along the x axis, and rotation around the x axis.

a) Compute the full transform matrix ${}^{b}\boldsymbol{T}_{ee}$ from the base of the robot to its end-effector.

b) Assume we require the end-effector to be at position $[a, b, c]^T$, and we do not care about end-effector orientation.

- Show how to compute values for all the robot joints such that the end-effector achieves the desired position.
- How many solutions are there in the general case? If the number of solutions depends on certain characteristics of a, b and c, explain how and why.

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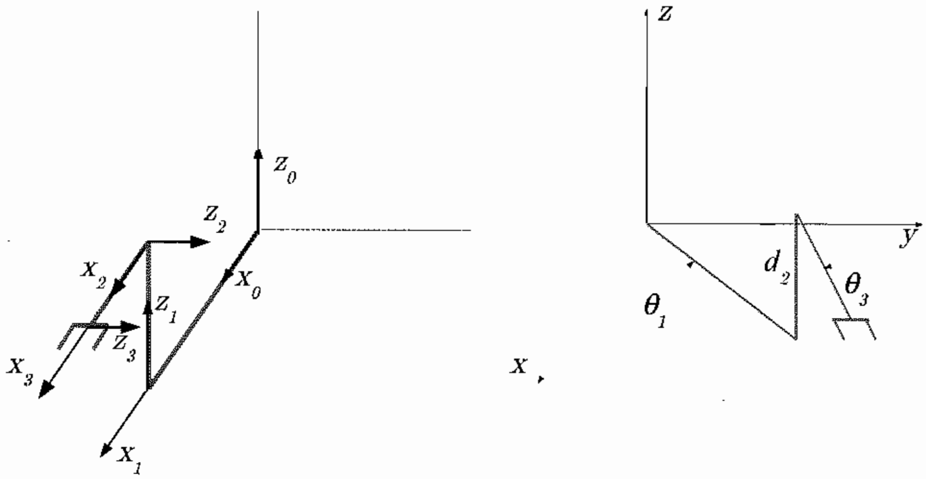
2 Differential Kinematics

Assume the following DH configuration for a robot:

i	θ	d	a	α
1	q_1	0	2	0
2	0	q_2	0	$-90°$
3	q_3	0	1	0



Note that, in the used convention, parameters θ_i, d_i, a_i and α_i define the transform from coordinate frame $\{i-1\}$ to coordinate frame $\{i\}$, as illustrated in the picture. These paremeters represent, in order, rotation around the z axis, translation along the z axis, translation along the x axis, and rotation around the x axis.

The translation part of the transform matrix ${}^b\boldsymbol{T}_{ee}$ from the base of the robot to its end-effector is:

$$ {}^b\boldsymbol{T}_{ee} = \left[\begin{array}{ccc|c} & & & C_1(2+C_3) \\ & \boldsymbol{R} & & S_1(2+C_3) \\ & & & d_2 - S_3 \\ \hline 0 & 0 & 0 & 1 \end{array} \right] $$

using the shorthand notation $C_i = \cos(\theta_i)$ and $S_i = \sin(\theta_i)$.

a) Compute the manipulator Jacobian $\boldsymbol{J}$.

b) Find a robot configuration in which the Jacobian has a nullspace of dimensionality 1. Find the basis vector of this nullspace (note that the basis vector should have norm 1).

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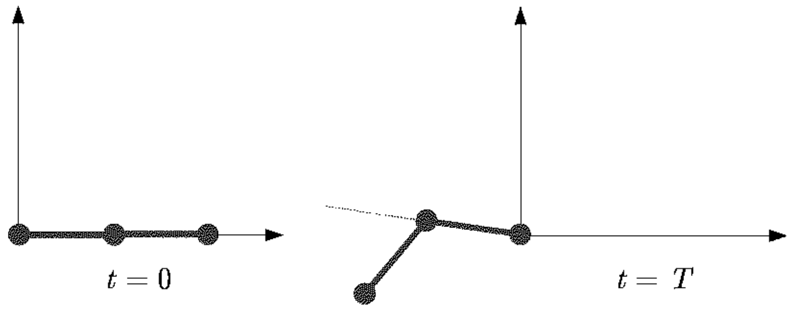
3 Trajectory Computation and Dynamics

A planar 2-link manipulator must execute the trajectory segment shown in the image below:



At $t = 0$, the robot is at rest with joint values $\boldsymbol{q} = [0, 0]^T$. At $t = T$ the robot must be at rest with joint values $\boldsymbol{q} = [3, 1]^T$. The maximum absolute velocity of both joints is 1 rad/s. The maximum absolute acceleration of the first (proximal) joint is 0.5 rad/s^2; the maximum acceleration of the second (distal) joint is 1 rad/s^2.

a) Assume that the robot allows discontinuous joint acceleration profiles (in other words, any desired joint acceleration can be achieved instantaneously). Furthermore, we require that both joints start moving simultaneously and also reach their final rest poses simultaneously; other than that, there are no synchronization requirements between the joints during execution of the trajectory. Under these assumptions, what is the shortest time in which the robot can execute the requested trajectory segment?

b) Under the same assumptions as above, plot the acceleration, velocity, and position profiles (as functions of time) for both joints during execution of the trajectory segment.

c) At the beginning of the trajectory ($t = 0$) what are the joint torques needed to achieve the desired joint accelerations? Assume the following dynamic properties:

- inertia matrix $\boldsymbol{M} = \begin{bmatrix} 6 & 2 \\ 2 & 1 \end{bmatrix}$
- Coriolis and centripetal matrix $\boldsymbol{C} = \boldsymbol{0}^{2x2}$
- gravity vectory $\boldsymbol{G} = [4, 1]^T$
- joints are considered frictionless
- no other external force is applied to the end-effector

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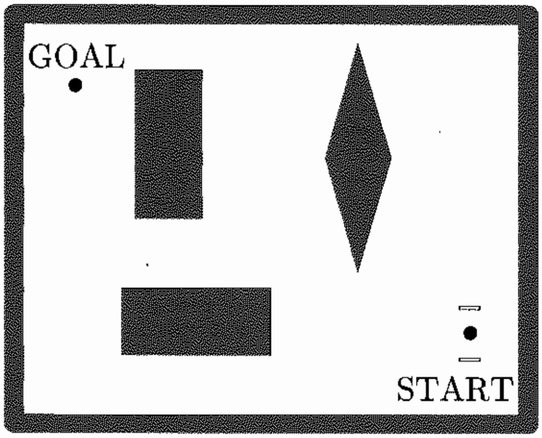
4 Motion Planning

Consider the following scenario. A mobile robot with a circular footprint is operating in an environment described by a polygonal map, as shown in the image below. A goal point is provided by an external operator; the robot must plan a path to reach the goal point starting from its current position.



a) Describe the steps that must be taken to compute a path to the goal using Dijkstra's algorithm. This includes both preparatory steps (e.g. modifications to the map) and the steps that comprise the algorithm itself.

b) How is the A* algorithm different from Dijkstra's algorithm, what are the additional requirements in order to be able to plan a path using A*, and what are the advantages of using A* over Dijkstra's algorithm?

c) Describe a stochastic motion planning algorithm that can be applied in this sitation (e.g. RRT, PRM, etc.). What are the advantages and disadvantages of using such an approach over using A* or Dijkstra's algorithms?