7.48

MANUFACTURING
Doctoral Qualifying Examination, May 2016
Mechanical Engineering Department, Columbia University

Problem 1

a) Calculate the electrochemical equivalent k of (i) Titanium in trivalent form; and (ii) an alloy consisting of 62% Nickel in divalent form and 38%Aluminum. Also calculate the electrochemical machinability K_v for the metal and the alloy. Assume the current efficiency η is 0.95. Assume the percentages given above are weight percentages. What is the physical meaning of the electrochemical equivalent and the electrochemical machinability? If the applied voltage U is 12V, calculate specific energy e. (See material data at the end of the Manufacturing section)

b) It is known that the inter-electrode gap variation under constant tool feed rate (1D case) can be expressed as $\frac{dS}{dt} = K_v \kappa \frac{U - \Delta U}{S} - V_f$, where S - gap size, K_v – EC machinability, κ - electrolyte conductivity, U – applied voltage, ΔU - total surface over-voltage (voltage drop from the source to the anode), and V_f – tool feed rate. Assuming the tool is stationary, solve the above equation and obtain an expression of the gap S as an explicit function of time t, if the $U - \Delta U$ is (i) constant and (ii) e^{-2t}. The initial gap is S_0, that is, when $t = 0$, $S = S_0$.

c) In an electrochemical machining process, if an anode surface is described by $y_a = 0.5\sqrt{1 - x_a^2}$, (i) determine the anode surface normal $\vec{N}_a$ and the angle between the anode surface normal $\vec{N}_a$ and the y axis, that is, $\cos\alpha = \cos(\overrightarrow{N_a}, \vec{j})$, and (ii) determine the expression for the coordinates of tool electrode profile (x_e, y_e), assuming $S_f = K_v \kappa \frac{U - \Delta U}{V_f}$ is known.

MANUFACTURING
Doctoral Qualifying Examination, May 2016
Mechanical Engineering Department, Columbia University

Problem 2

a) It is known that cooling is a major requirement in gas lasers such as CO_2 lasers (wavelength = 10.6 μm). Two types of cooling scheme are commonly used: (1) slow flow and (2) fast axial flow. What is the main cooling mechanism for each type?

b) In a slow flow CO_2 laser, the laser cavity is cylindrical with the aperture radius of 7 *mm*, the waste heat uniformly generated within the cavity space is $2.3 \times 10^6 W/m^3$, the coolant temperature is 20°C, and the maximum allowable gas temperature in the cavity is 250°C. Determine the waste heat generated per unit length of the cavity space, thermal conductivity of the gas mixture (assuming it is a constant), and the temperature at radius of 2.4 *mm*. For the same laser, if laser power is 110 *W* and the wall plug efficiency is 11%, determine the cavity length required to produce such power and the Fresnel number of the laser cavity.

c) The surface of semi-infinite Ni solid is uniformly heated by $F_0 = 5.5\text{x}10^9$ W/m^2. Heat conduction can be considered as one-dimensional with no convection, radiation and melting. Determine (i) the time t_m for the surface ($z = 0$) to reach the melting temperature, and (ii) when $t = t_m$ μs, let $z/2\sqrt{\alpha t} = 1$ and determine the value of z.

7.50

MANUFACTURING
Doctoral Qualifying Examination, May 2016
Mechanical Engineering Department, Columbia University

Problem 3

a) A thick Ti plate is welded using a laser source of 510W at a speed of 50mm/s. If the absorption of the laser power is 65% and the ambient temperature is $20°C$, determine the peak temperature on the plate surface at a distance of 0.75 mm from the center line along which the laser travels.

b) If surface temperature lower than $650°C$ is not considered to affect the Ti plate, determine the heat affected zone size.

c) Determine the surface cooling rate at 1,8 mm behind the laser spot along the center line.

MANUFACTURING
Doctoral Qualifying Examination, May 2016
Mechanical Engineering Department, Columbia University

Problem 4

a) A low carbon steel sheet metal is bent using a 680W laser. The sheet thickness is 2.1 mm. If the beam radius is 2.4 mm, scanning is done at a rate of 70 mm/s and a surface reflection coefficient of 55% is assumed, determine the bending mechanism dominating the process. Coefficient of thermal expansion, $\alpha_{th} = 12.6 \times 10^{-6}\ /K$. For other material data, use those of Iron's.

b) Determine the time of laser material interaction at irradiated surface (dwelling time) and the time to thermally conduct through the thickness.

c) Determine the bend angle, α_B, assuming a two-beam model.

d) Determine the temperature rise in the top portion of the sheet metal, assuming a two-beam model.

MANUFACTURING
Doctoral Qualifying Examination, May 2016
Mechanical Engineering Department, Columbia University

Useful data

Faraday's constant $F = 96{,}500\ coulomb\ (or\ A \cdot sec)$
Planck's constant $h_p = 6.625 \times 10^{-34}\ J \cdot sec$
Velocity of light in vacuum $c = 3 \times 10^{8}\ m/sec$
Boltzmann's constant $k_B = 1.38 \times 10^{-23} J/K$
$1eV = 1.6 \times 10^{-19} J$

Material	Atomic weight	Valence	Density	
	g		g/cm3	lb/in3
Aluminum	26.98	3	2.71	0.09799
Copper	63.546	1	8.97	0.32436
		2		
Iron	55.847	2	7.86	0.28422
		3		
Lead	207.19	2	11.36	0.41078
		4		
Nickel	58.71	2	8.92	0.32255
		3		
Platinum	195.09	2	21.47	0.77636
		4		
Titanium	47.9	3	4.52	0.16344
		4		

Material	Material properties							
	ρ (kg m^{-3})	L_f (kJ kg^{-1})	L_v (kJ kg^{-1})	C_p (J kg^{-1} °C^{-1})	T_m (°C)	T_v (°C)	K (W m^{-1} K^{-1})	α ($\times 10^{-5}$ m^2 s^{-1})
Tungsten	19,300	185	4,020	140	3,410	5,930	164	6.07
Aluminium	2,700	397	9,492	900	660	2,450	226	9.30
Iron	7,870	275	6,362	460	1,536	3,000	50	1.38
Titanium	4,510	437	9,000	519	1,668	3,260	19	0.81
304 stainless steel	8,030	≈ 300	6,500	500	1,450	3,000	20	0.5

Nickel: $T_m = 1{,}455°\text{C}, \alpha = 2.298 \times 10^{-5} m^2 s^{-1}, K = 90.9 W m^{-1} K^{-1}$

MANUFACTURING
Doctoral Qualifying Examination, January 2016
Mechanical Engineering Department, Columbia University

Problem 1

a) In an electrochemical machining process, if an anode surface is described by $z = 0.3\sqrt{1-0.4y^2}$, determine the anode surface normal $\vec{n}_a$ and the angle between the anode surface normal $\vec{n}_a$ and the z axis, that is, $\cos\alpha = \cos(\vec{n}_a, \vec{k})$.

b) Calculate the electrochemical equivalent of (i) Nickel in trivalent form; and (ii) an alloy consisting of 55% Copper in divalent form and 45% Aluminum. Also calculate the electrochemical machinability for the metal and the alloy. Assume the current efficiency is 0.9. Assume the percentages given above are weight percentages. What is the physical meaning of the electrochemical equivalent and the electrochemical machinability? Why the current efficiency is less than one? (See data at the end of the Manufacturing portion of the Qualifying Exam)

c) The governing equation of the one-dimensional inter-electrode gap G under constant tool feed rate V_f is $\frac{dG}{dt} = V_l - V_f = \frac{\kappa K_v (U - \Delta U)}{G} - V_f$, where V_l – linear material removal rate, κ - electrolyte conductivity, K_v - electrochemical machinability, U – applied voltage, and ΔU - total surface over-voltage.
(i) Determine the equilibrium gap size; (ii) If instead the tool is stationary and $U - \Delta U = 2.1$ V, solve the above equation and obtain an explicit expression of the gap G as a function of time t, for initial gap G_0; and (iii) Repeat (ii) if the tool is still stationary but $U - \Delta U = e^{-2.5t}$ V.

MANUFACTURING
Doctoral Qualifying Examination, January 2016
Mechanical Engineering Department, Columbia University

Problem 2

a) An incoming laser beam has wavelength 0.9 μm and waist radius $w_0 = 3$ mm at $z = 0.0$ m. If the beam radius $w(z)$ at $z = 5$ m is measured as 4.15 mm, determine its M^2 value.

b) To design a Nd:YAG laser (wavelength =1.06 μm) cavity having Fresnel number of 17, determine the cavity length L and aperture radius a, if a/L must equal to 0.0038. What is the physical meaning of the Fresnel number?

c) Briefly explain laser cavity filling and stability. If a laser cavity has length, L, and both the front and back mirrors are concave ($R_1>0$, $R_2>0$) and $R_1 = 2R_2$, determine the range of R_1 and R_2 if the cavity is to be stable.

MANUFACTURING
Doctoral Qualifying Examination, January 2016
Mechanical Engineering Departmeut, Columbia University

Problem 3

A low carbon steel plate is to be bent with a 350 *W* laser, whose beam spot size is 8 *mm*. The plate is 110 *mm* wide and 190 *mm* long with a surface refection of 55%. The scanning is along the sheet width direction with velocity of 14 *mm/s*. The modulus of elasticity, $E = 208$ *GPa*, and linear coefficient of thermal expansion, $\alpha_{th} = 12.6 \times 10^{-6}$ */K*. For other material properties of the low carbon steel, please use the ones for iron given in an attached table. After the first scan, the sheet is bent by 0.4°.

a) Determine the sheet thickness.

b) In this process, which forming mechanism is dominant? Why?

c) Determine the bending moment that is induced during the process.

d) Determine the temperature rise between the top and bottom layer of the sheet.

MANUFACTURING
Doctoral Qualifying Examination, January 2016
Mechanical Engineering Department, Columbia University

Problem 4

a) A thick Ti plate is welded using a laser source of 500W at a speed of 50mm/s. If the absorption of the laser power is 65% and the ambient temperature is 20°*C*, determine the peak temperature on the plate surface at a distance of 0.8 mm from the center line along which the laser travels.

b) If surface temperature lower than 660°*C* is not considered to affect the Ti plate, determine the heat affected zone size.

c) Determine the surface cooling rate at 2mm behind the laser spot along the center line.

MANUFACTURING
Doctoral Qualifying Examination, January 2016
Mechanical Engineering Department, Columbia University

Useful data

Faraday's constant $F = 96{,}500\ coulomb\ (or\ A \cdot sec)$
Planck's constant $h_p = 6.625 \times 10^{-34}\ J \cdot sec$
Velocity of light in vacuum $c = 3 \times 10^{8}\ m/sec$
Boltzmann's constant $k_B = 1.38 \times 10^{-23} J/K$
$1eV = 1.6 \times 10^{-19} J$

Material	**Atomic weight**	**Valence**	**Density**	
	g		**g/cm3**	**lb/in3**
Aluminum	26.98	3	2.71	0.09799
Copper	63.546	1	8.97	0.32436
		2		
Iron	55.847	2	7.86	0.28422
		3		
Lead	207.19	2	11.36	0.41078
		4		
Nickel	58.71	2	8.92	0.32255
		3		
Platinum	195.09	2	21.47	0.77636
		4		
Titanium	47.9	3	4.52	0.16344
		4		

Material	Material properties ρ (kg m^{-3})	L_f (kJ kg^{-1})	L_v (kJ kg^{-1})	C_p (J kg^{-1} °C^{-1})	T_m (°C)	T_v (°C)	K (W m^{-1} K^{-1})	α ($\times 10^{-5}$ m^2 s^{-1})
Tungsten	19,300	185	4,020	140	3,410	5,930	164	6.07
Aluminium	2,700	397	9,492	900	660	2,450	226	9.30
Iron	7,870	275	6,362	460	1,536	3,000	50	1.38
Titanium	4,510	437	9,000	519	1,668	3,260	19	0.81
304 stainless steel	8,030	≈ 300	6,500	500	1,450	3,000	20	0.5

Nickel: $T_m = 1{,}455°\text{C}, \alpha = 2.298 \times 10^{-5} m^2 s^{-1}, K = 90.9 W m^{-1} K^{-1}$

Doctoral Qualifying Examination, January 2015
Mechanical Engineering Department, Columbia University
MANUFACTURING

Problem 1

a) Calculate the electrochemical equivalent k of (i) Titanium in trivalent form; and (ii) an alloy consisting of 62% Nickel in divalent form and 38%Aluminum. Also calculate the electrochemical machinability K_v for the metal and the alloy. Assume the current efficiency η is 0.95. Assume the percentages given above are weight percentages. What is the physical meaning of the electrochemical equivalent and the electrochemical machinability? If the applied voltage U is 12V, calculate specific energy e. (See material data at the end of the Manufacturing section)

b) It is known that the inter-electrode gap variation under constant tool feed rate (1D case) can be expressed as $\frac{dS}{dt} = K_v \kappa \frac{U - \Delta U}{S} - V_f$, where S - gap size, K_v – EC machinability, κ - electrolyte conductivity, U – applied voltage, ΔU - total surface over-voltage (voltage drop from the source to the anode), and V_f – tool feed rate. Assuming the tool is stationary, solve the above equation and obtain an expression of the gap S as an explicit function of time t, if the $U - \Delta U$ is (i) constant and (ii) e^{-2t}. The initial gap is S_0, that is, when $t = 0$, $S = S_0$.

c) In an electrochemical machining process, if an anode surface is described by $y_a = 0.5\sqrt{1 - x_a^2}$, (i) determine the anode surface normal $\vec{N}_a$ and the angle between the anode surface normal $\vec{N}_a$ and the z axis, that is, $\cos\alpha = \cos(\vec{N}_a, \vec{k})$, and (ii) determine the expression for the coordinates of tool electrode profile (x_e, y_e), assuming $S_f = K_v \kappa \frac{U - \Delta U}{V_f}$ is known.

Doctoral Qualifying Examination, January 2015
Mechanical Engineering Department, Columbia University
MANUFACTURING

Problem 2

a) A low carbon steel sheet metal is bent using a 680W laser. The sheet thickness is 2.1 mm. If the beam radius is 2.4 mm, scanning is done at a rate of 70 mm/s and a surface reflection coefficient of 55% is assumed, determine the bending mechanism dominating the process. Coefficient of thermal expansion, $\alpha_{th} = 12.6 \times 10^{-6}$ /K. For other material data, use those of Iron's.

b) Determine the time of laser material interaction at irradiated surface (dwelling time) and the time to thermally conduct through the thickness.

c) Determine the bend angle, α_B, assuming a two-layer model.

d) Determine the temperature rise in the top portion of the sheet metal, assuming a two-layer model.

643

Doctoral Qualifying Examination, January 2015
Mechanical Engineering Department, Columbia University
MANUFACTURING

Problem 3

a) It is known that cooling is a major requirement in gas lasers such as CO_2 lasers (wavelength = 10.6 μm). Two types of cooling scheme are commonly used: (1) slow flow and (2) fast axial flow. What is the main cooling mechanism for each type?

b) In a slow flow CO_2 laser, the laser cavity is cylindrical with the aperture radius of 7 mm, the waste heat uniformly generated within the cavity space is $2.3 \times 10^6 W/m^3$, the coolant temperature is 20°C, and the maximum allowable gas temperature in the cavity is 250°C. Determine the waste heat generated per unit length of the cavity space, thermal conductivity of the gas mixture (assuming it is a constant), and the temperature at radius of 2.4 mm. For the same laser, if laser power is 110 W and the wall plug efficiency is 11%, determine the cavity length required to produce such power and the Fresnel number of the laser cavity.

c) The surface of semi-infinite Ni solid is uniformly heated by $F_0 = 5.5\text{x}10^9$ W/m^2. Heat conduction can be considered as one-dimensional with no convection, radiation and melting. Determine (i) the time t_m for the surface ($z = 0$) to reach the melting temperature, and (ii) when $t = t_m$ µs, let $z/2\sqrt{\alpha t} = 1$ and determine the value of z.

$$T_m = 1455^\circ C$$
$$\alpha = 2298 \times 10^{-5} \frac{m^2}{s}$$
$$K = 90.9 \frac{W}{mK}$$

Doctoral Qualifying Examination, January 2015
Mechanical Engineering Department, Columbia University
MANUFACTURING

Useful data

Faraday's constant $F = 96{,}500\ coulomb\ (or\ A \cdot sec)$
Planck's constant $h_p = 6.625 \times 10^{-34}\ J \cdot sec$
Velocity of light in vacuum $c = 3 \times 10^{8}\ m/sec$
Boltzmann's constant $k_B = 1.38 \times 10^{-23} J/K$
$1eV = 1.6 \times 10^{-19} J$

Material	Atomic weight	Valence	Density	
	g		g/cm3	lb/in3
Aluminum	26.98	3	2.71	0.09799
Copper	63.546	1	8.97	0.32436
		2		
Iron	55.847	2	7.86	0.28422
		3		
Lead	207.19	2	11.36	0.41078
		4		
Nickel	58.71	2	8.92	0.32255
		3		
Platinum	195.09	2	21.47	0.77636
		4		
Titanium	47.9	3	4.52	0.16344
		4		

Material	Material properties							
	ρ ($\mathrm{kg\,m^{-3}}$)	L_f ($\mathrm{kJ\,kg^{-1}}$)	L_v ($\mathrm{kJ\,kg^{-1}}$)	C_p ($\mathrm{J\,kg^{-1}\,°C^{-1}}$)	T_m (°C)	T_v (°C)	K ($\mathrm{W\,m^{-1}\,K^{-1}}$)	α ($\times 10^{-5}\ \mathrm{m^2\,s^{-1}}$)
Tungsten	19,300	185	4,020	140	3,410	5,930	164	6.07
Aluminium	2,700	397	9,492	900	660	2,450	226	9.30
Iron	7,870	275	6,362	460	1,536	3,000	50	1.38
Titanium	4,510	437	9,000	519	1,668	3,260	19	0.81
304 stainless steel	8,030	≈ 300	6,500	500	1,450	3,000	20	0.5

MANUFACTURING

Doctoral Qualifying Examination, January 2014
Mechanical Engineering, Columbia University

Problem 1

a) In a one-dimensional electrochemical machining process with a stationary tool, it is known that the initial gap between the tool and workpiece is 2.5 mm, the coefficient of electrochemical machinability K_v is 4.39 mm^3/Amin, the electrolyte conductivity κ is 0.5 A/Vmm, the voltage at the cathode is 0 V, and ΔU at the anode is zero. Determine the gap z_a at t=7 min if the voltage at the anode is constant at $U = 1.2$ V.

b) Determine the gap z_a at t=7 min if the voltage at the anode is $U = 0.14t$ V, where t is time in minutes.

c) In an electrochemical machining process, if an anode surface is described by $z = 0.4\sqrt{1-y^2}$, determine the anode surface normal $\vec{N}_a$ and the angle between the anode surface normal $\vec{N}_a$ and the z axis, that is, $\cos\alpha = \cos(\vec{N}_a, \vec{k})$.

Problem 2

a) If a CO_2 laser is of the slow flow type, determine the minimal aperture radius a, such that the limiting temperature $T_{lim} = 250°C$ will not be exceeded in the cavity. It is known that the waste heat $Q = 5.2x10^6$ W/m^3, coolant temperature $T_c = 20°C$, and gas thermal conductivity $k = 0.14$ W/mK.

b) Determine the r value at which the temperature is 40% of the limiting value of 250°C.

c) Determine its wall plug efficiency if the laser generates 175W power and has a cavity length of 3.4 m.

Problem 3

a) A low carbon steel sheet metal (similar to iron in properties) is bent using an 750W laser. The sheet thickness is 0.9 mm, width 85 mm, and length 110 mm. If the beam radius is 1.75 mm, scanning is done at a rate of 45 mm/s along the width of the sheet metal and a surface reflection coefficient of 60% is assumed, determine the bending mechanism dominating the process. (Modulus of Elasticity, $E = 208\ GPa$, and linear coefficient of thermal expansion, $\alpha_{th} = 12.6 \times 10^{-6}\ /K$.)

b) Determine, after the first pass the bend angle, α_B.

c) Determine the temperature rise in the top portion of the sheet metal.

d) Determine the induced bending moment, M.

Useful data

Faraday's constant $F = 96{,}500\ coulomb\ (or\ A \cdot sec)$
Planck's constant $h_p = 6.625 \times 10^{-34}\ J \cdot sec$
Velocity of light in vacuum $c = 3 \times 10^{8}\ m/sec$
Boltzmann's constant $k_B = 1.38 \times 10^{-23} J/K$
$1eV = 1.6 \times 10^{-19} J$

Material	Atomic weight	Valence	Density	
	g		g/cm3	lb/in3
Aluminum	26.98	3	2.71	0.09799
Copper	63.546	1	8.97	0.32436
		2		
Iron	55.847	2	7.86	0.28422
		3		
Lead	207.19	2	11.36	0.41078
		4		
Nickel	58.71	2	8.92	0.32255
		3		
Platinum	195.09	2	21.47	0.77636
		4		
Titanium	47.9	3	4.52	0.16344
		4		

Material	Material properties							
	ρ (kg m^{-3})	L_f (kJ kg^{-1})	L_v (kJ kg^{-1})	C_p (J kg^{-1} °C^{-1})	T_m (°C)	T_v (°C)	K (W m^{-1} K^{-1})	α ($\times 10^{-5}$ m^2 s^{-1})
Tungsten	19300	185	4020	140	3410	5930	164	6.07
Aluminium	2700	397	9492	900	660	2450	226	9.30
Iron	7870	275	6362	460	1536	3000	50	1.38
Titanium	4510	437	9000	519	1668	3260	19	0.81
Stainless steel (304)	8030	~300	6500	500	1450	3000	20	0.50

560

Doctoral Qualifying Examination, January 2013
Mechanical Engineering, Columbia University
MANUFACTURING

Problem 1

a) Briefly describe the difference between spontaneous emission and stimulated emission.
b) It is know that, under thermal equilibrium, $N_2/N_1 = 0.85$ and $A_e/B = 1.45 \times 10^{-21}$ (Js/m^3), where N_2 and N_1 are number of atoms per unit volume at higher and lower energy levels, respectively, and A_e and B are probability per unit time of spontaneous and stimulated emission, respectively. Determine temperature T and frequency ν associated with the atomic transition.
c) If the stimulated emission probability B is 6.2×10^{31} m^3/Js^2, determine how long it takes for N_2 to reduce to 20% of its initial value.
d) In an optical resonator, where the reflectivities of the back and front mirrors are $R_1 = 100\%$ and $R_2 = 33\%$, the length is $L = 0.58$m and the medium absorbs 6% of a laser light. If it is known that the laser light is increased by 53% after a round trip. Determine the small signal gain coefficient β. What does β account for?

Doctoral Qualifying Examination, January 2013
Mechanical Engineering, Columbia University
MANUFACTURING

Problem 2

a) If a laser drilling process of stainless steel 304 can be described by one-dimensional heat conduction model, please state at least three assumptions under which the model is largely valid.
b) If it is known that the absorbed laser intensity is a constant of 525,000 W/cm^2, determine how long it takes for the steel surface to reach its vaporization temperature.
c) At that time t, what is the temperature at depth of $2\sqrt{\alpha t}$, where α is the diffusivity of the steel?

Doctoral Qualifying Examination, January 2013
Mechanical Engineering, Columbia University
MANUFACTURING

Problem 3

A low carbon steel plate is to be bent with a 350 *W* laser, whose beam spot size is 8 *mm*. The plate is 110 *mm* wide and 190 *mm* long with a surface refection of 55%. The scanning is along the sheet width direction with velocity of 14 *mm/s*. The modulus of elasticity, $E = 208$ *GPa,* and linear coefficient of thermal expansion, $\alpha_{th} = 12.6 \times 10^{-6}$ /*K*. After the first scan, the sheet is bent by 0.4°.

a) Determine the sheet thickness.
b) In this process, which forming mechanism is dominant? Why?
c) Determine the bending moment that is induced during the process.
d) Determine the temperature rise between the top and bottom layer of the sheet.

563

Doctoral Qualifying Examination, January 2013
Mechanical Engineering, Columbia University
MANUFACTURING

Useful data

Boltzmann's constant $k_B = 1.38 \times 10^{-23}$ J/K
Planck's constant $h_p = 6.625 \times 10^{-34}$ J·sec
Velocity of light in vacuum $c = 3 \times 10^{8}$ m/sec
1 eV = 1.602×10^{-19} J

Material	Material properties							
	ρ (kg m^{-3})	L_f (kJ kg^{-1})	L_v (kJ kg^{-1})	C_p (J kg^{-1} °C^{-1})	T_m (°C)	T_v (°C)	K (W m^{-1} K^{-1})	α ($\times 10^{-5}$ m^2 s^{-1})
Tungsten	19300	185	4020	140	3410	5930	164	6.07
Aluminium	2700	397	9492	900	660	2450	226	9.30
Iron	7870	275	6362	460	1536	3000	50	1.38
Titanium	4510	437	9000	519	1668	3260	19	0.81
Stainless steel (304)	8030	~300	6500	500	1450	3000	20	0.50

Error Function[1]

w	erf w	w	erf w	w	erf w
0.00	0.00000	0.36	0.38933	1.04	0.85865
0.02	0.02256	0.38	0.40901	1.08	0.87333
0.04	0.04511	0.40	0.42839	1.12	0.88679
0.06	0.06762	0.44	0.46622	1.16	0.89910
0.08	0.09008	0.48	0.50275	1.20	0.91031
0.10	0.11246	0.52	0.53790	1.30	0.93401
0.12	0.13476	0.56	0.57162	1.40	0.95228
0.14	0.15695	0.60	0.60386	1.50	0.96611
0.16	0.17901	0.64	0.63459	1.60	0.97635
0.18	0.20094	0.68	0.66378	1.70	0.98379
0.20	0.22270	0.72	0.69143	1.80	0.98909
0.22	0.24430	0.76	0.71754	1.90	0.99279
0.24	0.26570	0.80	0.74210	2.00	0.99532
0.26	0.28690	0.84	0.76514	2.20	0.99814
0.28	0.30788	0.88	0.78669	2.40	0.99931
0.30	0.32863	0.92	0.80677	2.60	0.99976
0.32	0.34913	0.96	0.82542	2.80	0.99992
0.34	0.36936	1.00	0.84270	3.00	0.99998

[1] The Gaussian error function is defined as

$$\text{erf}\, w \equiv \frac{2}{\sqrt{\pi}} \int_0^w e^{-v^2}\, dv$$

The complementary error function is defined as

$$\text{erfc}\, w \equiv 1 - \text{erf}\, w$$

Useful data

Faraday's constant F = 96,500 coulomb (or A·sec)
Planck's constant $h = 6.625\text{x}10^{-34}$ J·sec
Velocity of light in vacuum $c = 3\text{x}10^{8}$ m/sec
$1\text{eV} = 1.6\text{x}10^{-19}\text{J}$

Material	Atomic weight	Valence	Density	
	g		g/cm3	lb/in3
Aluminum	26.98	3	2.71	0.09799
Copper	63.546	1	8.97	0.32436
		2		
Iron	55.847	2	7.86	0.28422
		3		
Lead	207.19	2	11.36	0.41078
		4		
Nickel	58.71	2	8.92	0.32255
		3		
Platinum	195.09	2	21.47	0.77636
		4		
Titanium	47.9	3	4.52	0.16344
		4		

Material	Material properties							
	ρ (kg m^{-3})	L_f (kJ kg^{-1})	L_v (kJ kg^{-1})	C_p (J kg^{-1} °C^{-1})	T_m (°C)	T_v (°C)	K (W m^{-1} K^{-1})	α ($\times10^{-5}$ m^2 s^{-1})
Tungsten	19300	185	4020	140	3410	5930	164	6.07
Aluminium	2700	397	9492	900	660	2450	226	9.30
Iron	7870	275	6362	460	1536	3000	50	1.38
Titanium	4510	437	9000	519	1668	3260	19	0.81
Stainless steel (304)	8030	~300	6500	500	1450	3000	20	0.50

MANUFACTURING

Doctoral Qualifying Examination, January 2012
Mechanical Engineering, Columbia University

Problem 1

a) In an electrochemical machining process, if an anode surface is described by $z = 0.3\sqrt{1-0.4y^2}$, determine the anode surface normal $\vec{n}_a$ and the angle between the anode surface normal $\vec{n}_a$ and the z axis, that is, $\cos\alpha = \cos(\vec{n}_a, \vec{k})$.

b) Calculate the electrochemical equivalent of (i) Nickel in trivalent form; and (ii) an alloy consisting of 55% Copper in divalent form and 45% Aluminum. Also calculate the electrochemical machinability for the metal and the alloy. Assume the current efficiency is 0.9. Assume the percentages given above are weight percentages. What is the physical meaning of the electrochemical equivalent and the electrochemical machinability? Why the current efficiency is less than one? (See data at the end of the Manufacturing portion of the Qualifying Exam)

c) The governing equation of the one-dimensional inter-electrode gap G under constant tool feed rate V_f is $\frac{dG}{dt} = V_l - V_f = \frac{\kappa K_v (U - \Delta U)}{G} - V_f$, where V_l – linear material removal rate, κ - electrolyte conductivity, K_v - electrochemical machinability, U – applied voltage, and ΔU - total surface over-voltage.
(i) Determine the equilibrium gap size; (ii) If instead the tool is stationary and $U - \Delta U = 2.1$ V, solve the above equation and obtain an explicit expression of the gap G as a function of time t, for initial gap G_0; and (iii) Repeat (ii) if the tool is still stationary but $U - \Delta U = e^{-2.5t}$ V.

Problem 2

a) An incoming laser beam has wavelength 0.9 μm and waist radius $w_0 = 3$ mm at $z = 0.0$ m. If the beam radius $w(z)$ at $z = 5$ m is measured as 4.15 mm, determine its M^2 value.

b) To design a Nd:YAG laser (wavelength =1.06 μm) cavity having Fresnel number of 17, determine the cavity length L and aperture radius a, if a/L must equal to 0.0038. What is the physical meaning of the Fresnel number?

c) Briefly explain laser cavity filling and stability. If a laser cavity has length, L, and both the front and back mirrors are concave ($R_1>0$, $R_2>0$) and $R_1 = 2R_2$, determine the range of R_1 and R_2 if the cavity is to be stable.

Problem 3

a) In a CO_2 laser, what physical phenomena do the energy levels correspond to? Give two examples. What is the purpose of adding N_2 and He? Define the wall plug efficiency.

b) If the CO_2 laser is of the slow flow type, determine the minimal aperture radius a, such that the limiting temperature T_{lim} = 250°C will not be exceeded in the cavity. It is known that the waste heat Q =6.5x10^6 W/m^3, coolant temperature T_c = 20°C, and gas thermal conductivity k = 0.14 W/mK. Also determine the r value at which the temperature is 2/3 of the limiting value of 250°C.

c) For a laser vaporization drilling process of titanium plates, calculate (i) the time it takes for the surface to reach the vaporization temperature, assuming one-dimensional heat flow, no convection or heat generation, and the absorbed laser intensity is 1.7x10^{10} W/m^2; and (ii) the time it takes for the laser to penetrate the 1.9 mm plate thickness. (See data at the end of the manufacturing section).

MANUFACTURING

Doctoral Qualifying Examination, January 2011
Mechanical Engineering, Columbia University

Problem 1

a) Calculate the electrochemical equivalent of (i) Aluminum; and (ii) an alloy consisting of 55% Copper in divalent form and 45% Aluminum (the percentages given are weight percentages). Also calculate the electrochemical machinability for the metal and the alloy. Assume the current efficiency is 0.8. What is the physical meaning of the electrochemical equivalent and the electrochemical machinability? Why the current efficiency is less than one? (See data at the end of the Manufacturing portion of this exam paper)

b) If the applied voltage U is 12 V and total current at anode I is 785 A, calculate volumetric material removal rate Q_v, mass removal rate Q_m, and specific energy e.

c) It is known that the inter-electrode gap variation under constant tool feed rate (1D case) can be expressed as $\frac{dG}{dt} = V_l - V_f = \frac{\kappa K_v (U - \Delta U)}{G} - V_f$

where G - gap size
V_l – linear removal rate
V_f – tool feed rate
κ - electrolyte conductivity
K_v - electrochemical machinability
U – applied voltage
ΔU - total surface over-voltage

(i) Determine the equilibrium gap size; and (ii) if the tool is now stationary and $U - \Delta U = e^{-t}$, solve the above equation and obtain an explicit expression of the gap G as a function of time t, for initial gap G_0.

Problem 2

a) An incoming laser beam has wavelength 1.06 μm and waist radius w_0 = 4.2 mm at z = 0.0 m. If the beam radius $w(z)$ at z = 6.5 m is measured as 4.5 mm, determine its M^2 value.

b) For the transverse electromagnetic mode of a laser beam, TEM_{pl}, explain what p and l values mean and the physical cause of the phenomena behind these values. If the laser beam indicated in (a) is TEM_{20} and is generated from a cavity whose aperture radius is 2.1 mm, determine the cavity length.

c) Briefly explain laser cavity filling and stability. If a laser cavity has length, L, and both the front and back mirrors are concave (R>0) with equal radius, R, determine the range of R if the cavity is to be stable.

Problem 3

a) A laser light of wavelength of 355 nm irradiates from the normal direction at a metal surface exposed in air. (i) If the extinction coefficient of the metal is 10.5, determined the skin depth of the light; and (ii) if the complex index of refraction of the metal is $n_c = 2.3\text{-}j10.5$, determine the reflectance of the light from the metal surface.

b) In CO_2 lasers, what physical phenomena do the different energy levels correspond to? Give two examples. What is the purpose of adding N_2 and He? Define the quantum efficiency and the wall plug efficiency.

c) For a laser vaporization drilling process of titanium plates, calculate (i) the time it takes for the surface to reach the vaporization temperature, assuming one-dimensional heat flow, no convection or heat generation, and the absorbed laser intensity is $1.5\text{x}10^{10}$ W/m^2; and (ii) the time it takes for the laser to penetrate the 2 mm plate thickness.

Useful data

Faraday's constant F = 96,500 coulomb (or A·sec)
Planck's constant $h = 6.625 \times 10^{-34}$ J·sec
Velocity of light in vacuum $c = 3 \times 10^{8}$ m/sec
$1 \text{eV} = 1.6 \times 10^{-19}$ J

Material	Atomic weight	Valence	Density	
	g		g/cm3	lb/in3
Aluminum	26.98	3	2.71	0.09799
Copper	63.546	1	8.97	0.32436
		2		
Iron	55.847	2	7.86	0.28422
		3		
Lead	207.19	2	11.36	0.41078
		4		
Nickel	58.71	2	8.92	0.32255
		3		
Platinum	195.09	2	21.47	0.77636
		4		
Titanium	47.9	3	4.52	0.16344
		4		

Material	Material properties							
	ρ (kg m^{-3})	L_f (kJ kg^{-1})	L_v (kJ kg^{-1})	C_p (J kg^{-1} °C^{-1})	T_m (°C)	T_v (°C)	K (W m^{-1} K^{-1})	α ($\times 10^{-5}$ m^2 s^{-1})
Tungsten	19300	185	4020	140	3410	5930	164	6.07
Aluminium	2700	397	9492	900	660	2450	226	9.30
~~Iron~~	7870	275	6362	460	1536	3000	50	1.38
Titanium	4510	437	9000	519	1668	3260	19	0.81
Stainless steel (304)	8030	~300	6500	500	1450	3000	20	0.50

MANUFACTURING

Doctoral Qualifying Examination, January 2010
Mechanical Engineering, Columbia University

Problem 1

a) Calculate the electrochemical equivalent of (i) Nickel in trivalent form; and (ii) an alloy consisting of 55% Copper in divalent form and 45% Aluminum. Also calculate the electrochemical machinability for the metal and the alloy. Assume the current efficiency is 0.9. Assume the percentages given above are weight percentages. What is the physical meaning of the electrochemical equivalent and the electrochemical machinability? Why the current efficiency is less than one? (See data at the end of the Manufacturing portion of the Qualifying Exam)

b) In an one-dimensional electrochemical machining process with a stationary tool, it is known that the initial gap between the tool and workpiece is 2.5 mm, the coefficient of electrochemical machinability K_v is 3.9 mm^3/Amin, the electrolyte conductivity κ is 0.63 A/Vmm, and the voltage at the cathode is 0 V and ΔU at the anode is assumed to be zero. Determine the gap z_a at $t = 3.8$ min if the voltage at the anode is constant at U=1.2V.

c) Determine the gap z_a at $t = 3.8$ min if the voltage at the anode is $U = e^{-t}$ V, where t is time in minutes. Also determine the equilibrium gap size under this applied voltage. The other conditions are the same as in b).

Problem 2

a) To design a Nd:YAG laser (wavelength =1.06 μm) cavity having Fresnel number of 18, determine the cavity length L and aperture radius a, if a/L must equal to 0.004. What is the physical meaning of the Fresnel number?

b) A circular laser beam of diameter 7 mm and wavelength 850 nm is focused by a lens with focus length 90 mm. If the beam is of TEM_{00} (Transverse Electromagnetic Mode), determine the diffraction-limited spot size and depth of field.

c) An incoming laser beam has wavelength 10 μm and waist radius $w_0 = 3$ mm at $z = 0.0$ m. If the beam radius $w(z)$ at $z = 7$ m is measured as 3.5 mm, determine its M^2 value.

Problem 3

a) In a CO_2 laser, what physical phenomena do the energy levels correspond to? Give two examples. What is the purpose of adding N_2 and He? Define the wall plug efficiency.

b) If the CO_2 laser is of the slow flow type, determine the minimal aperture radius a, such that the limiting temperature $T_{lim} = 250°C$ will not be exceeded in the cavity. It is known that the waste heat $Q = 6.5 \times 10^6$ W/m^3, coolant temperature $T_c = 20°C$, and gas thermal conductivity $k = 0.14$ W/mK. Also determine the r value at which the temperature is 2/3 of the limiting value of 250°C.

c) For a laser vaporization drilling process of aluminum plates, calculate (i) the time it takes for the surface to reach the vaporization temperature, assuming one-dimensional heat flow, no convection or heat generation, and the absorbed laser intensity is 2×10^{10} W/m^2; and (ii) the time it takes for the laser to penetrate the 1.5-mm plate thickness (See data at the end of the manufacturing section).

Useful data

Faraday's constant F = 96,500 coulomb (or A·sec)
Planck's constant $h = 6.625\text{x}10^{-34}$ J·sec
Velocity of light in vacuum $c = 3\text{x}10^{8}$ m/sec
$1\text{eV} = 1.6\text{x}10^{-19}\text{J}$

Material	Atomic weight	Valence	Density	
	g		g/cm3	lb/in3
Aluminum	26.98	3	2.71	0.09799
Copper	63.546	1	8.97	0.32436
		2		
Iron	55.847	2	7.86	0.28422
		3		
Lead	207.19	2	11.36	0.41078
		4		
Nickel	58.71	2	8.92	0.32255
		3		
Platinum	195.09	2	21.47	0.77636
		4		
Titanium	47.9	3	4.52	0.16344
		4		

Material	Material properties							
	ρ (kg m^{-3})	L_f (kJ kg^{-1})	L_v (kJ kg^{-1})	C_p (J kg^{-1} °C^{-1})	T_m (°C)	T_v (°C)	K (W m^{-1} K^{-1})	α ($\times 10^{-5}$ m^2 s^{-1})
Tungsten	19300	185	4020	140	3410	5930	164	6.07
Aluminium	2700	397	9492	900	660	2450	226	9.30
~~Iron~~	7870	275	6362	460	1536	3000	50	1.38
Titanium	4510	437	9000	519	1668	3260	19	0.81
Stainless steel (304)	8030	~300	6500	500	1450	3000	20	0.50

427

MANUFACTURING

Problem 1:

a) In an electrochemical machining process, if an anode surface is described by $z = 0.4\sqrt{1-0.5y^2}$, determine the anode surface normal $\vec{n}_a$ and the angle between the anode surface normal $\vec{n}_a$ and the z axis, that is, $\cos\alpha = \cos(\vec{n}_a, \vec{k})$.

b) Calculate the electrochemical equivalent k and coefficient of electrochemical machinability K_v for divalent nickel, assuming the current efficiency η is 90%. (See data at the end of the manufacturing section). Describe the physical meaning of k and K_v.

c) The governing equation of the one-dimensional inter-electrode gap G under constant tool feed rate V_f is $\dfrac{dG}{dt} = V_l - V_f = \dfrac{\kappa K_v(U - \Delta U)}{G} - V_f$, where V_l – linear material removal rate, κ - electrolyte conductivity, K_v - electrochemical machinability, U – applied voltage, and ΔU - total surface over-voltage.

   i. Determine the equilibrium gap size

   ii. If instead the tool is stationary and $U - \Delta U = 1.5$ V, solve the above equation and obtain an explicit expression of the gap G as a function of time t, for initial gap G_0

   iii. Repeat (ii) if the tool is still stationary but $U - \Delta U = e^{-3t}$ V.

428

MANUFACTURING

Problem 2:

a) An incoming laser beam has wavelength 1.1 μm and waist radius $w_0 = 4$ mm at $z = 0.0$ m. If the beam radius $w(z)$ at $z = 6$ m is measured as 4.66 mm, determine its M^2 value.

b) For the transverse electromagnetic mode of a laser beam, TEM_{pl}, explain what p and l values mean and the physical cause of the phenomena behind these values. If the laser beam indicated in (a) is TEM_{30} and is generated from a cavity whose aperture radius is 2.5 mm, determine the cavity length.

c) Briefly explain laser cavity filling and stability. If a laser cavity has length, L, and both the front and back mirrors are concave ($R_1>0$, $R_2>0$) and $R_1 = 2R_2$, determine the range of R_1 and R_2 if the cavity is to be stable.

429

MANUFACTURING

Problem 3:

a) A 300 W slow-axial-flow CO_2 laser has a cavity length L=0.5 m and aperture radius a=11 mm. Determine the waste heat Q in W/m^3 and wall plug efficiency, given limiting temperature T_{lim} = 250°C, coolant temperature T_c = 20°C, and gas thermal conductivity k = 0.14 W/m°C.

b) For fusion cutting of titanium using an inert gas, the specific energy is about 14 J/mm^2. (i) Given laser power P = 850 W, determine cutting speed v in mm/s for workpiece thickness t = 3 mm and (ii) Sketch v vs. t for the given power.

c) For a laser vaporization drilling process of aluminum plates, calculate (i) the time it takes for the surface to reach the vaporization temperature, assuming one-dimensional heat flow, no convection or heat generation, and the absorbed laser intensity is 2x10^{10} W/m^2; and (ii) the time it takes for the laser to penetrate the 2.5-mm plate thickness (See data at the end of the manufacturing section).

430

USEFUL DATA FOR MANUFACTURING QUESTIONS

Faraday's constant F = 96,500 coulomb (or A·sec)
Planck's constant $h = 6.625 \times 10^{-34}$ J·sec
Velocity of light in vacuum $c = 3 \times 10^{8}$ m/sec
$1 \text{eV} = 1.6 \times 10^{-19}$ J

Material	Atomic weight	Valence	Density	
	g		g/cm^3	lb/in^3
Aluminum	26.98	3	2.71	0.09799
Copper	63.546	1	8.97	0.32436
		2		
Iron	55.847	2	7.86	0.28422
		3		
Lead	207.19	2	11.36	0.41078
		4		
Nickel	58.71	2	8.92	0.32255
		3		
Platinum	195.09	2	21.47	0.77636
		4		
Titanium	47.9	3	4.52	0.16344
		4		

Material	Material properties							
	ρ (kg m^{-3})	L_f (kJ kg^{-1})	L_v (kJ kg^{-1})	C_p (J kg^{-1} °C^{-1})	T_m (°C)	T_v (°C)	K (W m^{-1} K^{-1})	α ($\times 10^{-5}$ m^2 s^{-1})
Tungsten	19300	185	4020	140	3410	5930	164	6.07
Aluminium	2700	397	9492	900	660	2450	226	9.30
Iron	7870	275	6362	460	1536	3000	50	1.38
Titanium	4510	437	9000	519	1668	3260	19	0.81
Stainless steel (304)	8030	~300	6500	500	1450	3000	20	0.50

MANUFACTURING

Doctoral Qualifying Examination, January 2008
Mechanical Engineering, Columbia University

Problem 1

a) In an electrochemical machining process, if an anode surface is described by $x^2 + 4z^2 - 1 = 0$, determine the anode surface normal $\vec{n}_a$.

b) For a nickel iron consisting of 6% of nickel in trivalent and the rest of iron in bivalent, calculate electrochemical equivalent k and the coefficient of electrochemical machinability K_v, assuming that each element dissolves independently and simultaneously during the electrochemical machining. Assume the current efficiency is 0.85. If the applied voltage U is 22 V and total current at anode I is 740 A, calculate mass removal rate Q_m, and specific energy e (See data at the end of the Manufacturing portion of the Qualifying Exam).

c) The governing equation of the one-dimensional inter-electrode gap under constant tool feed rate is $\frac{dG}{dt} = V_l - V_f = \frac{\kappa K_v (U - \Delta U)}{G} - V_f$, where G - gap size, V_l – linear material removal rate, V_f – tool feed rate, $\varkappa$ - electrolyte conductivity, K_v - electrochemical machinability, U – applied voltage, and ΔU - total surface over-voltage.

(i) Determine the equilibrium gap size,

(ii) If instead the tool is stationary and $U - \Delta U = 2.0$ V, solve the above equation and obtain an explicit expression of the gap G as a function of time t, for initial gap G_0, and

(iii) Repeat (ii) if the tool is still stationary but $U - \Delta U = 0.5t$ V.

3.96

Problem 2

a) A circular laser beam of diameter 8 mm and wavelength 355 nm is focused by a lens with focus length of 25 mm. Determine the diffraction-limited spot size and depth of field,

(i) if the beam is of Gaussian, that is, in TEM_{00} (Transverse electromagnetic mode),

(ii) if the beam is in TEM_{21}, or

(iii) if the beam is known to have a M^2 value of 2.3.

b) An incoming laser beam has wavelength 10 μm and waist radius $w_0 = 3$ mm at $z = 0.0$ m. If the beam radius $w(z)$ at $z = 7$ m is measured as 3.5 mm, determine its M^2 value.

c) For a TEM_{40} laser beam generated from a cavity whose aperture radius is 2 mm, determine the cavity length.

Problem 3

a) For laser fusion cutting of 304 stainless steel involving an inert gas, the specific energy is about 8 J/mm^2. If the cutting speed is chosen as 30 mm/s, determine the laser power in W required cutting a 2-mm thick plate. Assume heat conduction is negligible.

b) If the same laser was used for vaporization drilling of the same 304 stainless steel plate, what penetration velocity in m/s can it achieve? Assume the absorption coefficient is 0.4 for both the fusion and vaporization cases, heat conduction is negligible, and initial temperature is 20°C.

c) For laser forming of aluminum plates of 1.5 mm thick with a scanning speed v = 30 mm/s and a beam diameter D =14 mm or 3 mm, what is the likely forming mechanism for each case? Justify your answer quantitatively.

_Useful data

Faraday's constant F = 96,500 coulomb (or A·sec)
Planck's constant $h = 6.625 \times 10^{-34}$ J·sec
Velocity of light in vacuum $c = 3 \times 10^{8}$ m/sec
1eV = 1.6×10^{-19}J

Material	Atomic weight	Valence	Density	
	g		g/cm3	lb/in3
Aluminum	26.98	3	2.71	0.09799
Copper	63.546	1	8.97	0.32436
		2		
Iron	55.847	2	7.86	0.28422
		3		
Lead	207.19	2	11.36	0.41078
		4		
Nickel	58.71	2	8.92	0.32255
		3		
Platinum	195.09	2	21.47	0.77636
		4		
Titanium	47.9	3	4.52	0.16344
		4		

Material	Material properties							
	ρ (kg m^{-3})	L_f (kJ kg^{-1})	L_v (kJ kg^{-1})	C_p (J kg^{-1} °C^{-1})	T_m (°C)	T_v (°C)	K (W m^{-1} K^{-1})	α ($\times 10^{-5}$ m^2 s^{-1})
Tungsten	19300	185	4020	140	3410	5930	164	6.07
Aluminium	2700	397	9492	900	660	2450	226	9.30
~~Iron~~	7870	275	6362	460	1536	3000	50	1.38
Titanium	4510	437	9000	519	1668	3260	19	0.81
Stainless steel (304)	8030	~300	6500	500	1450	3000	20	0.50

Problem 1

a) Describe the physical meaning of electrochemical equivalent k and the coefficient of electrochemical machinability K_v. If an alloy consisting of 35% trivalent titanium and the rest divalent nickel, calculate its electrochemical equivalent k assuming that each element dissolves independently and simultaneously during electrochemical machining. Also calculate its coefficient of electrochemical machinability K_v assuming current efficiency is 0.8. (See data at the end of the Manufacturing portion of the Qualifying Exam).

b) If the applied voltage U is 14 V and total current at anode I is 825 A, calculate volumetric material removal rate Q_v, mass removal rate Q_m, and specific energy e.

c) In an one-dimensional electrochemical machining process with a stationary tool, it is known that the initial gap between the tool and workpiece is 2.75 mm, the coefficient of electrochemical machinability K_v is 4 mm^3/Amin, the electrolyte conductivity κ is 0.65 A/Vmm, and the voltage at the cathode is 0 V and ΔU at the anode is assumed to be zero. Determine the gap z_a at $t = 4$ min if the voltage at the anode is constant at U=1.25V.

d) Determine the gap z_a at $t = 4$ min if the voltage at the anode is $U = e^{-2t}$ V, where t is time in minutes. Also determine the equilibrium gap size under this applied voltage. The other conditions are the same as in c)

MANUFACTURING

Doctoral Qualifying Examination, January 2007
Mechanical Engineering, Columbia University

Problem 2

a) An incoming laser beam has wavelength 1.06 μm and waist radius $w_0 = 4.5$ mm. If one measured the beam radius $w(z) = 4.77$ mm at $z = 6.5$ m from the waist, determine its M^2 value.

b) For the transverse electromagnetic mode of a laser beam, TEM_{pl}, explain what p and l values mean and the physical cause of the phenomena behind these values.

c) Briefly explain laser cavity filling and stability. If a laser cavity has length, L, and both the front and back mirrors are concave (R>0) with equal radius, R, determine the range of R values for a stable cavity.

Problem 3

a) In a CO_2 laser, what do the energy levels correspond to? Give two examples. What is the purpose of adding N_2 and *He*? Define the wall plug efficiency.

b) If the CO_2 laser is of the slow flow type, determine the minimal aperture radius a, such that the limiting temperature T_{lim} = 250°C will not be exceeded in the cavity. It is known that the waste heat Q =6.5x10^6 W/m^3, coolant temperature T_c = 20°C, and gas thermal conductivity k = 0.14 W/mK. Also determine the r value at which the temperature is 2/3 of the limiting value of 250°C.

c) If such a laser was used for vaporization cutting of tungsten with a penetration velocity V = 1.25 m/s, determine the absorbed laser density F_o in W/m^2 (assuming heat conduction negligible). See data at the end of the Manufacturing portion of the Qualifying Exam.

_Useful data

Faraday's constant $F = 96{,}500$ coulomb (or A·sec)
Planck's constant $h = 6.625\text{x}10^{-34}$ J·sec
Velocity of light in vacuum $c = 3\text{x}10^{8}$ m/sec
$1\text{eV} = 1.6\text{x}10^{-19}\text{J}$

Material	Atomic weight	Valence	Density	
	g		g/cm3	lb/in3
Aluminum	26.98	3	2.71	0.09799
Copper	63.546	1	8.97	0.32436
		2		
Iron	55.847	2	7.86	0.28422
		3		
Lead	207.19	2	11.36	0.41078
		4		
Nickel	58.71	2	8.92	0.32255
		3		
Platinum	195.09	2	21.47	0.77636
		4		
Titanium	47.9	3	4.52	0.16344
		4		

Material	Material properties							
	ρ (kg m^{-3})	L_f (kJ kg^{-1})	L_v (kJ kg^{-1})	C_p (J kg^{-1} °C^{-1})	T_m (°C)	T_v (°C)	K (W m^{-1} K^{-1})	α ($\times10^{-5}$ m^2 s^{-1})
Tungsten	19300	185	4020	140	3410	5930	164	6.07
Aluminium	2700	397	9492	900	660	2450	226	9.30
Iron	7870	275	6362	460	1536	3000	50	1.38
Titanium	4510	437	9000	519	1668	3260	19	0.81
Stainless steel (304)	8030	~300	6500	500	1450	3000	20	0.50

MANUFACTURING

Doctoral Qualifying Examination, January 2006
Mechanical Engineering, Columbia University

Problem 1:

a) Calculate the electrochemical equivalent of (i) Aluminum; and (ii) an alloy consisting of 56% Copper in divalent form and 44% Aluminum. Also calculate the electrochemical machinability for the metal and the alloy. Assume the current efficiency is 0.8. Assume the percentages given above are weight percentages. What is the physical meaning of the electrochemical equivalent and the electrochemical machinability? Why the current efficiency is less than one? (See data at the end of this exam paper)

b) It is known that the inter-electrode gap variation under constant tool feed rate (1D case) can be expressed as $\frac{dG}{dt} = V_l - V_f = \frac{\kappa K_v (U - \Delta U)}{G} - V_f$

where G - gap size
V_l – linear removal rate
V_f – tool feed rate
κ - electrolyte conductivity
K_v - electrochemical machinability
U – applied voltage
ΔU - total surface over-voltage

(i) Determine the equilibrium gap size; and (ii) if the tool is now stationary and $U - \Delta U = e^{-t}$, solve the above equation and obtain an explicit expression of the gap G as a function of time t, for initial gap G_0.

MANUFACTURING

Doctoral Qualifying Examination, January 2006
Mechanical Engineering, Columbia University

Problem 2:

a) A circular laser beam of diameter 10 mm and wavelength 800 nm is focused by a lens with focus length 100 mm. If the beam is of TEM_{00} (Transverse electromagnetic mode), determine the diffraction-limited spot size and depth of field.

b) If the laser cavity is to be modified to have TEM_{50}, what value should the aperture radius be if the cavity length is 1 meter? What will be the diffraction-limited spot size now?

c) If the cavity length remains to be 1 meter, both the front and back mirrors of the laser cavity are concave (R_1>0, R_2>0), and $R_1 = 2R_2$, determine the range of R_1 and R_2 for the cavity to be stable.

MANUFACTURING

Doctoral Qualifying Examination, January 2006
Mechanical Engineering, Columbia University

Problem 3:

a) A laser light of wavelength of 450 nm irradiates from the normal direction at a metal surface exposed in air. (i) If the extinction coefficient of the metal is 11, determined the skin depth of the light; and (ii) if the complex index of refraction of the metal is $n_c = 2.5\text{-}j11$, determine the reflectance of the light from the metal surface.

b) For a laser vaporization drilling process of titanium plates, calculate (i) the time it takes for the surface to reach the vaporization temperature, assuming one-dimensional heat flow, no convection or heat generation, and the absorbed laser intensity is 10^{10} W/m^2; and (ii) the time it takes for the laser to penetrate the 3-mm plate thickness, assuming no heat conduction and the absorbed laser intensity remains the same (See data at the end of this exam paper).

Useful data:
Faraday's constant F = 96,500 coulomb (or A·sec)

Metal	Atomic weight (g)	Valence	Density (g/cm^3)
Aluminum	26.98	3	2.71
Cadmium	112.40	2	8.67
Chromium	51.896	2, 3, 6	7.2
Cobalt	58.93	2, 3	8.92
Copper	63.546	1, 2	8.97

Material	Material properties							
	ρ (kg m^{-3})	L_f (kJ kg^{-1})	L_v (kJ kg^{-1})	C_p (J kg^{-1} °C^{-1})	T_m (°C)	T_v (°C)	K (W m^{-1} K^{-1})	α (×10^{-5} m^2 s^{-1})
Tungsten	19300	185	4020	140	3410	5930	164	6.07
Aluminium	2700	397	9492	900	660	2450	226	9.30
Iron	7870	275	6362	460	1536	3000	50	1.38
Titanium	4510	437	9000	519	1668	3260	19	0.81
Stainless steel (304)	8030	~300	6500	500	1450	3000	20	0.50

MANUFACTURING

Doctoral Qualifying Examination, January 2005
Mechanical Engineering, Columbia University

Problem 1

a) In an electrochemical machining process, if an anode surface is described by $z = 0.6\sqrt{1-y^2}$, determine the anode surface normal $\vec{n}_a$ and the angle between the anode surface normal $\vec{n}_a$ and the z axis, that is, $\cos\alpha = \cos(\vec{n}_a, \vec{k})$.

b) Calculate the electrochemical equivalent k for an alloy consisting of 45% Nickel in divalent and 55% Aluminum. What is the physical meaning of k? (See data at the end of this exam paper)

c) In an one-dimensional electrochemical machining process with a stationary tool, it is known that the initial gap between the tool and workpiece is 3 mm, the coefficient of electrochemical machinability K_v is 4.39 mm^3/Amin, the electrolyte conductivity κ is 0.5 A/Vmm, and the voltage at the cathode is 0 V and ΔU at the anode is assumed to be zero. Determine the gap z_a at $t = 10$ min if the voltage at the anode is
(i) $U = 1.1$ V, and (ii) $U = 0.2\,t$ V, where t is time in minutes.

Problem 2

a) To design a Nd:YAG laser (wavelength =1.06 μm) cavity having Fresnel number of 20, determine the cavity length L and aperture radius a, if a/L must equal to 0.005. What is the physical meaning of the Fresnel number?

b) A 100 W slow-axial-flow CO_2 laser has a cavity length L=3 m and aperture radius a=4 mm. Determine the waste heat Q in W/m^3 and wall plug efficiency, given limiting temperature T_{lim} = 240°C, coolant temperature T_c = 20°C, and gas thermal conductivity k = 0.14 W/m°C.

Problem 3

a) For fusion cutting of titanium using an inert gas, the specific energy is about 14 J/mm^2. (i) Given laser power P = 800 W, determine cutting speed v in mm/s for workpiece thickness t = 2 mm and (ii) Should the speed be halved if the thickness doubles?

b) Explain the physical meaning of Fourier number $\alpha D / vt^2$ (α is thermal diffusivity, D is beam spot size, v is scanning speed, and t is workpiece thickness) used in laser forming and explain how to use it to determine if the temperature gradient mechanism or the buckling mechanism is more dominant. For laser forming of aluminum plates of 2 mm thick with scanning speed v = 25 mm/s, determine the Fourier number value for (i) D = 10 mm, and (ii) D = 4 mm, and comment on dominant mechanism for each case.

Useful data:
Faraday's constant F = 96,500 coulomb (or A·sec)
Aluminum: atomic weight is 26.98g; valence is 3; thermal diffusivity is 97.46 mm^2/s
Nickel: atomic weight is 58.71g

MANUFACTURING

Doctoral Qualifying Examination, January 2004
Mechanical Engineering, Columbia University

Problem 1

a) In an electrochemical machining process, if an anode surface is described by $z = 0.5\sqrt{1-0.5y^2}$, determine the anode surface normal $\vec{n}_a$ and the angle between the anode surface normal $\vec{n}_a$ and the z axis, that is, $\cos\alpha = \cos(\vec{n}_a, \vec{k})$.

b) Calculate the electrochemical equivalent k and coefficient of electrochemical machinability K_v for aluminum, assuming the current efficiency η is 90%. What is the physical meaning of k and K_v? (Faraday's constant F = 96,500 coulomb (or A·sec); for aluminum, atomic weight is 26.98g, valence is 3, and density is 3.71g/cm^3.)

c) In an one-dimensional electrochemical machining process with a stationary tool, it is known that the initial gap between the tool and workpiece is 2 mm, the coefficient of electrochemical machinability K_v is 3 mm^3/Amin, the electrolyte conductivity κ is 0.5 A/Vmm, and the voltage at the cathode is 0 V and ΔU at the anode is assumed to be zero. Detemine the gap z_a at $t = 3$ min if the voltage at the anode is $U = t^2$ V, where t is time in minutes.

Problem 2

a) An incoming laser beam has wavelength 1.06 μm and diameter of 12.5 mm. It is focused by a lens with focal length of 125 mm. If the beam is of Gaussian, determine the diffraction limited spot size and the depth of field. What will be the diffraction limited spot size if the beam is of TEM_{11} type or if the beam has a M^2 value of 1.4 (TEM stands for transverse electromagnetic mode)

b) A CO_2 laser (λ = 10.6 μm) has a cavity length L=2 m. Determine the output aperture radius a if three fringes of the output laser beam are allowed. If the laser is of the slow flow type, determine the waste heat Q in W/m^3, given limiting temperature T_{lim} = 250°C, coolant temperature T_c = 20°C, and gas thermal conductivity k = 0.14 W/m°C. If this laser delivers power P = 110 W, determine its wall plug efficiency.

c) For vaporization cutting of stainless steel 304 with a penetration velocity V = 1.2 m/s, determine the absorbed laser density F_o in W/m^2 (assuming heat conduction negligible). See below for material properties.

Material	Material properties							
	ρ (kg m^{-3})	L_f (kJ kg^{-1})	L_v (kJ kg^{-1})	C_p (J kg^{-1} °C^{-1})	T_m (°C)	T_v (°C)	K (W m^{-1}K^{-1})	α (×10^{-5} m^2 s^{-1})
Tungsten	19300	185	4020	140	3410	5930	164	6.07
Aluminium	2700	397	9492	900	660	2450	226	9.30
Iron	7870	275	6362	460	1536	3000	50	1.38
Titanium	4510	437	9000	519	1668	3260	19	0.81
Stainless steel (304)	8030	~300	6500	500	1450	3000	20	0.50