HEAT TRANSFER
Doctoral Qualification Exam, May 2016
Mechanical Engineering Department, Columbia University

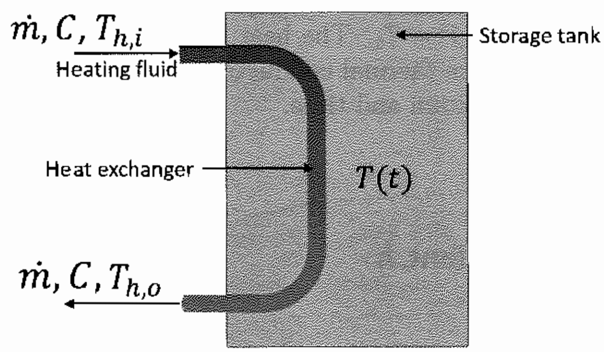


Figure 1: Problem 1.

Problem 1: A schematic of a part of a thermal storage tank is shown in Fig. 1. The volume of the storage tank is V_s, and the density and specific heat of the water in the tank are ρ_s and C_s respectively. The water in the tank is heated by a heat exchanger in the form of a thin-walled tube of length L and perimeter P. The supply temperature (mean temperature) $T_{h,i}$, mass flow rate $\dot{m}_h$, and specific heat C_h of the hot fluid flowing in the tube are kept constants. The average overall heat transfer coefficient between the heating fluid and the fluid in the tank is U. The temperature of the water in tank can be assumed to be independent of position. The tank is well-insulated so that losses to the ambient can be neglected. Answer the follwing questions:

1. Derive the equations for temperatnre $T(t)$ of the tank and the average temperature of the fluid, $T_h(x)$, along the length of the heat exchanger. You can make the simplifying assumption that $T_h(x)$ is not explicitly dependent on t. It is implicitly dependent on t through $T(t)$.

2. Show that $T_{h,i} - T_{h,o}(t) = [T_{h,i} - T(t)]\left[1 - \exp\left[-\dfrac{UPL}{\dot{m}_h C_h}\right]\right]$

3. Show that $[T(t) - T_{h,i}] = [T(0) - T_{h,i}]\, e^{-t/\tau}$, where $\tau = \dfrac{\rho_s C_s V_s}{(\dot{m}_h C_h)'}$, and $(\dot{m}_h C_h)' = (\dot{m}_h C_h)\left[1 - \exp\left[-\frac{UPL}{\dot{m}_h C_h}\right]\right]$.

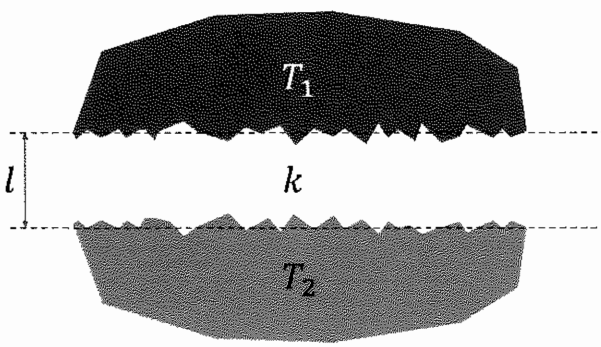


Figure 2: Problem 2

Problem 2: Find the thermal conduction heat transfer rate per unit area, q, between two half-spaces as shown in Fig. 2. The surfaces of the half-spaces are rough surfaces such that the nominal gap between the half-spaces is l. The protrusions Δ_1 and Δ_2 ($|\Delta_1|, |\Delta_2| \ll l$) are such that the areal averages $\langle\Delta_1\rangle$ and $\langle\Delta_2\rangle$ are zero, i.e., $\langle\Delta_1\rangle = \langle\Delta_2\rangle = 0$. The root mean square values $\langle\Delta_1^2\rangle$ and $\langle\Delta_2^2\rangle$ are uniform throughtout the surfaces. In addition, the surface roughness on the two surfaces are uncorrelated, i.e., $\langle\Delta_1\Delta_2\rangle = 0$. Determine q in terms of $k, l, \langle\Delta_1^2\rangle, \langle\Delta_2^2\rangle$.

Problem 3: Show that for fully developed Hagen-Poiseuille flow through a circular pipe of radius R of a highly viscous fuid, the energy equation reduces to:

$$k\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \mu\left(\frac{du}{dr}\right)^2 = 0$$

where k is the thermal conductivity, and μ is the dynamic viscosity. Find $u(r)$ and solve the above equation for $T(r)$ if the boundary of the tube ($r = R$) is maintained at a constant temperature T_w.

HEAT TRANSFER
Doctoral Qualification Exam, May 2016
Mechanical Engineering Department, Columbia University

Problem 4: The tip of a thermal fin ($x = L$ in Fig. 3) is heated by a time varying varying power of the form $P = P_o + \Delta P \sin(\omega t)$. The cross-sectional area (through which heat is conducted) is A_c and the circumference is C. The base ($x = 0$) is maintained at $T = T_b$. The heat transfer coefficient between the fin and the ambient fluid at temperature T_f is h. The thermal conductivity of the fin is k. Find the temperature distribution in the fin as a function of position and time.

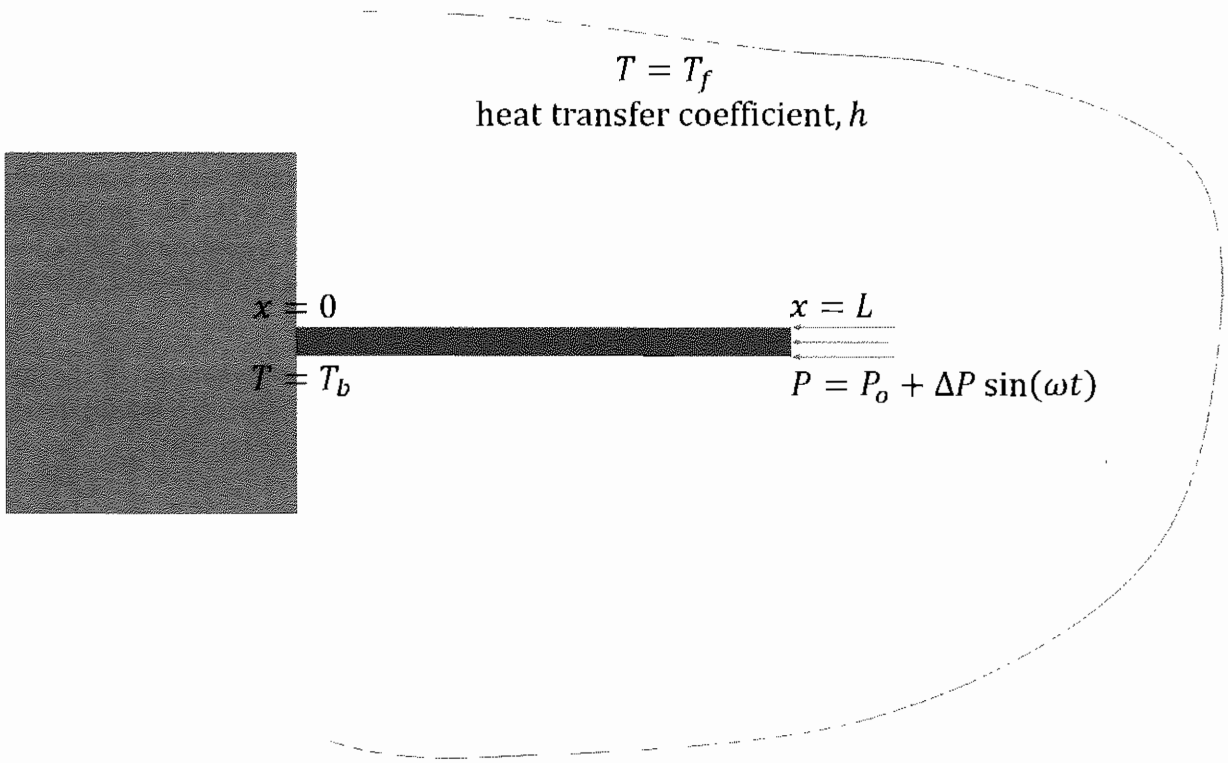


Figure 3: Problem 4

HEAT TRANSFER
Doctoral Qualifying Examination, January 2016
Mechanical Engineering Department, Columbia University

Problem 1: Fluid at temperature T_∞ and velocity U_∞ flows over a flat plate at uniform temperature T_w. The flow is laminar. If the Prandtl number Pr of the fluid satisfies the condition $Pr \ll 1$, then answer the following questions:

1. The curves marked A and B in the figure below correspond to boundary layers. Identify (by letter) the momentum boundary layer and the thermal boundary layer.
2. In the boundary layer approximation, conservation of energy reads as:
$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$
where u is the velocity in the x-direction and v is the velocity in the y-direction. What does this equation simply to in the limit $Pr \ll 1$? What are the boundary conditions?
3. Solve the PDE (appropriate for the $Pr \ll 1$ limit) and the boundary conditions to obtain the temperature distribution as a function of (x, y).
4. Find the local heat transfer coefficient between the flat plate and the fluid stream.
5. Express the heat transfer coefficient as a correlation between the Nusselt number, Reynolds number, and the Prandtl number.

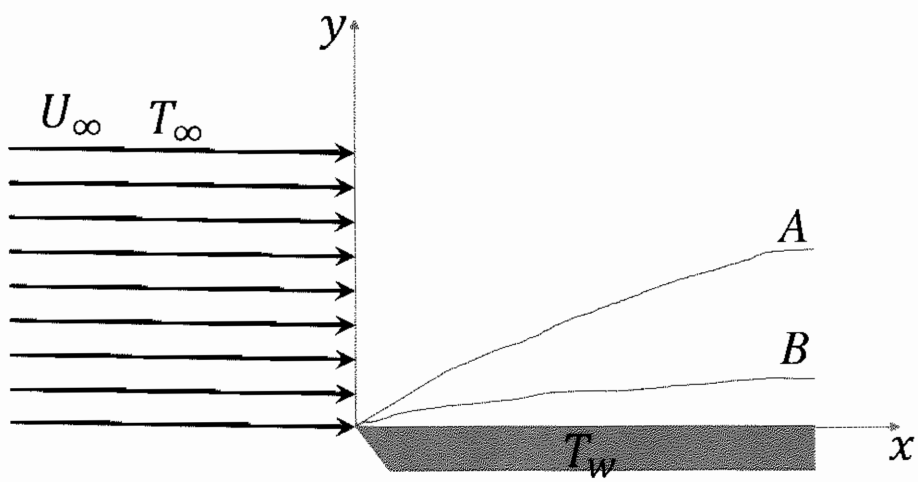
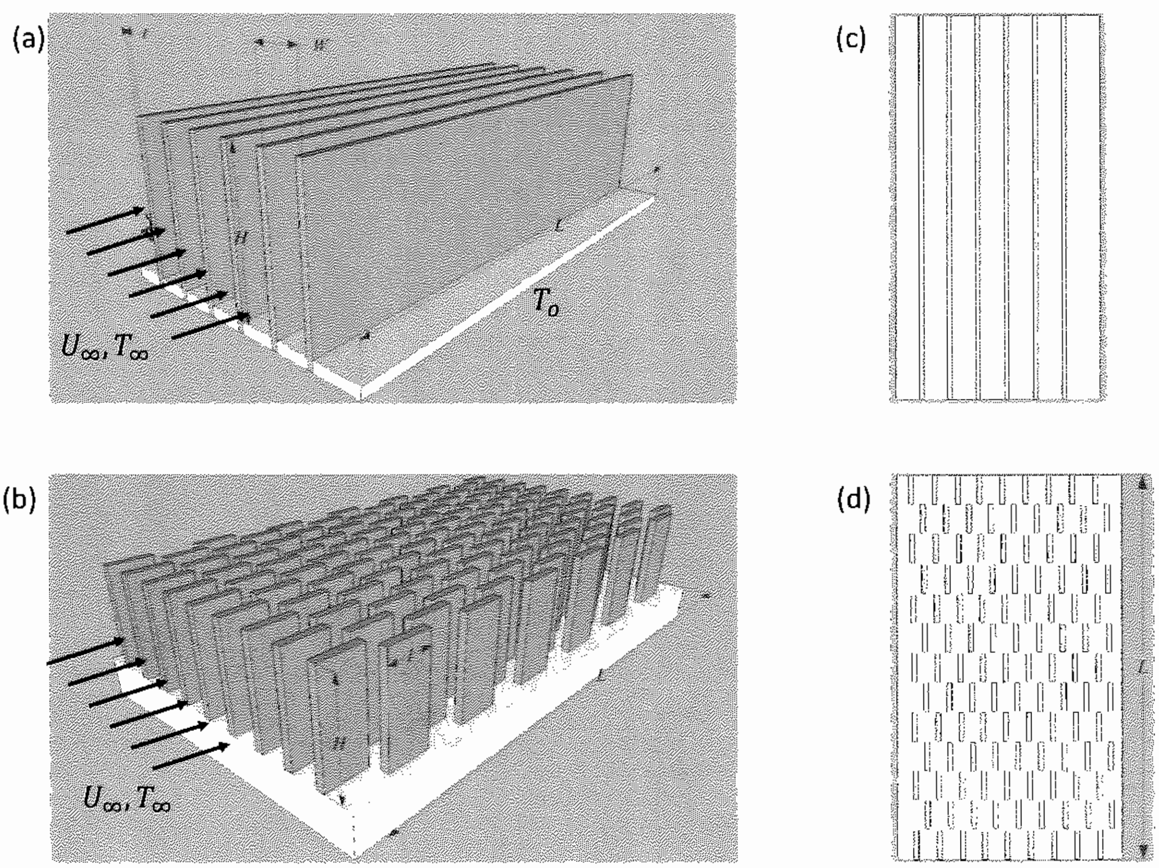


Figure 1: Flow over a flat plate at constant temperature. A and B correspond to boundary layers.

Useful integrals: $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-w^2}dw$, and $1 = \frac{2}{\sqrt{\pi}}\int_0^\infty e^{-w^2}dw$.

Problem 2: Compact heat exchangers have large surface-area-to-volume ratios, primarily because of the use of finned surfaces. The fluid-side heat exchanger surface for one such compact heat exchanger with fins shown in Fig. 2a (isometric view) and Fig. 2c (top view). The flow of air is along the direction indicated. The fins have a thickness t, height H, and length L. The flow is along the length of the fin. The distance between the centers of two adjacent fins is W. The flow is such that the boundary layer from a fin is much less than the spacing between two adjacent fins.

1. Under these assumptions, find the heat transfer rate per fin from the base to the fluid. All needed temperatures and velocities are shown in the figure.
2. One engineer proposes that it is possible to enhance the heat transfer with minimal changes to the fin design. She proposes the structure shown in Fig. 2b (and 2d), where $\frac{L}{l} = N$. First, explain qualitatively why the performance of the structure in Fig. 2b might be better than the original structure. Explain it with the aid of a simple graph (use whatever quantities you think are appropriate for the x and y axis). Whatever graph you draw, label the axes clearly. State any assumptions you made.
3. Compared to each fin in Fig. 2a, how much is the enhancement for the design in Fig. 2c?

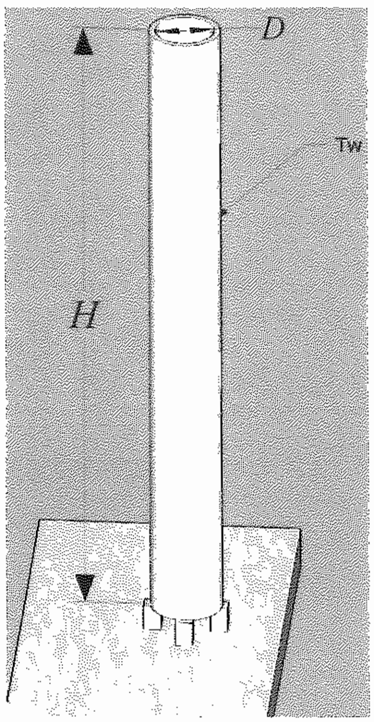


Problem 3: A schematic of a solar updraft tower is shown in Fig. 3. The walls of the cylindrical tower of internal diameter D and height H are at a temperature $T_w > T_\infty$ (ambient temperature). Because the air inside the cylindrical tower is heated, it rises due to buoyancy. The opening at the bottom is to ensure that air is supplied to keep the natural convection current going. Find the mass flow rate of air as a function of internal diameter D, height H, and other relevant properties such as ρ_∞, β (coefficient of thermal expansion), T_w, T_∞, μ (dynamic viscosity), and g (acceleration due to gravity). You can assume that the bulk mean temperature of air at any height is equal to T_w. You can also assume that the flow is fully developed.

Find the pressure gradient that is driving the flow and solve the appropriate simplification of the Navier-Stokes equation:

$$\rho\left(u\frac{\partial u}{\partial r} + v\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(ru)\right) + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(u\frac{\partial v}{\partial r} + v\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv)\right) + \frac{\partial^2 v}{\partial z^2}\right) - g$$



The z-axis is oriented along the axis of the tower.

Problem 4: Nuclear fuel elements (UO_2) in a pressurized water reactor (PWR) consist of a stack of cylindrical pellets within a hollow zircalloy cladding rod. The zircalloy rods are surrounded by pressurized water at 15 MPa and a temperature of approximately 320 °C. The volumetric heat generation rate within the fuel rods is approximately 350 $MW.m^{-3}$. The nominal diameter of the fuel pellets is 8.2 mm. The inner and outer diameter if the zircalloy cladding are 9.5 mm and 10.7 mm respectively. The gap between the fuel pellets and the zircalloy cladding is filled with helium gas. The thermal conductivities of the fuel elements, helium and cladding are 3 W/m/K, 0.142 W/m/K, and 21.5 W/m/K respectively. The heat transfer coefficient in between the cladding rod and the pressurized water is approximately 2000 $W/m^2/K$. Find the maximum temperature in the fuel element.

Doctoral Qualifying Examination, January 2015
Mechanical Engineering Department, Columbia University
HEAT TRANSFER

Problem 1. The governing equations of fluid flow and energy equations for a two-dimensional, steady state and incompressible flow with constant properties are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\underbrace{\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)}_{1}=\underbrace{\frac{-\partial P}{\partial x}}_{2}+\underbrace{\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)}_{3}$$

$$\underbrace{\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)}_{1'}=\underbrace{\frac{-\partial P}{\partial y}}_{2'}+\underbrace{\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)}_{3'}$$

$$\underbrace{\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)}_{4}=\underbrace{\mu\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right\}}_{5}+\underbrace{k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)}_{6}$$

a. Explain the physical meaning of terms 1 through 6 in the above equations (2 points).

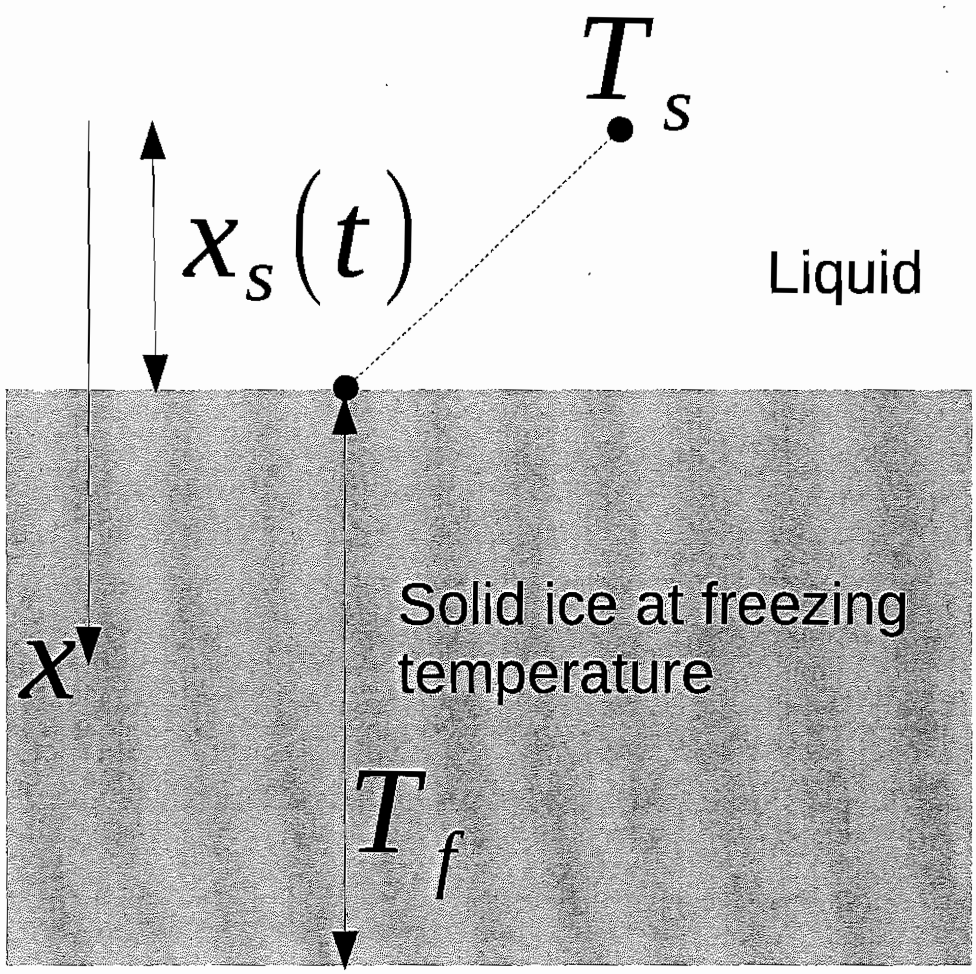
b. Using appropriate scaling (order of magnitude analysis) reduce those equations to the boundary layer equations (2 points).

c. Determine scales of the hydrodynamic boundary layer thickness (δ) , thermal boundary layer thickness (δ_T) and Nu number for $Pr >> 1$ and $Pr << 1$ (6 points).

Doctoral Qualifying Examination, January 2015
Mechanical Engineering Department, Columbia University
HEAT TRANSFER

Problem 2. The water (thermal conductivity k_w, latent heat of fusion h_{fl}) in a lake solidifies and is at the freezing temperature (T_f). At time $t=0$, the surface temperature, T_s, exceeds the freezing temperature, and remains at that value. Neglecting any effects due to transient heat conduction, find the thickness, $x_s(t)$, of the liquid water on the solid ice as a function of time. Assume that at time $t=0$, $x_s=0$.

State your assumptions in creating a thermal model to solve the problem. Derive the governing equation (1 pts) and boundary conditions clearly (5 pts). Correct solution for $x_s(t)$ is worth the remaining 4 pts.



Doctoral Qualifying Examination, January 2015
Mechanical Engineering Department, Columbia University
HEAT TRANSFER

Problem 3. The temperature distribution in a certain finite fin of circular cross-section can be approximated by:

$$\frac{T-T_\infty}{T_b-T_\infty}=\frac{e^{-3x}+e^{-6x}}{2}$$

where x, distance from the base, is in meters. The diameter of the fin is 5 mm and the conductivity of fin is 300 W/m K. The heat transfer coefficient to the surrounding air varies with position, i.e., $h(x)$.

a. Derive the differential equation for this fin (2 points).
b. Determine the heat transfer coefficient variation, $h(x)$, along the fin (4 points).
c. Determine the heat transfer from the fin to surroundings if $T_b=80^oC, T_\infty=25^oC$, and the fin length is 0.1 m (2 points).

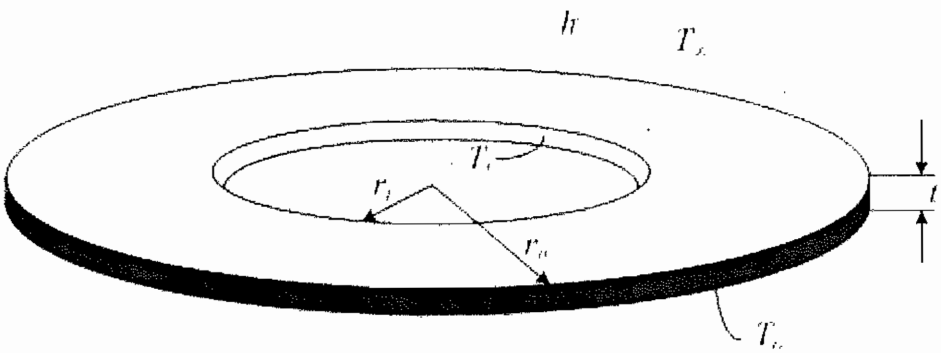
600

ADVANCED HEAT TRANSFER

Doctoral Qualifying Examination, January 2014
Mechanical Engineering, Columbia University

Problem 1:

A thin disk, as shown in the following figure, is kept at T_o at its outer edge and T_i at its inner edge. The air on both sides of the disk is at T_∞. The average heat transfer coefficient between the disk and air is h. The disk's inner and outer radii are r_i and r_o, respectively. Determine an algebraic expression for the temperature distribution along the radial direction of the disk. (10 points)



The following information may be helpful.

Modified Bessel Equation $x^2 \dfrac{d^2u}{dx^2} + x\dfrac{du}{dx} = \left(x^2 + n^2\right)u$

Two independent solutions are $I_n\left(x\right)$ and $K_n\left(x\right)$, the modified Bessel functions.

ADVANCE HEAT TRANSFER

Doctoral Qualifying Examination, January 2014
Mechanical Engineering, Columbia University

Problem 2:

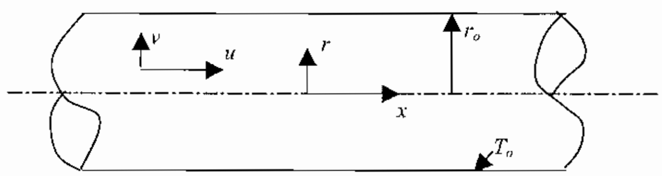
The energy equation for an axisymetric and incompressible laminar flow in a circular tube is given by:

$$\rho c_p\left(\overbrace{u\frac{\partial T}{\partial x}}^{1}+\overbrace{v\frac{\partial T}{\partial r}}^{2}\right)=k\left[\overbrace{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)}^{3}+\overbrace{\frac{\partial^2 T}{\partial x^2}}^{4}\right]+\overbrace{\mu\phi}^{5}$$

where

$$\varphi=2\left[\left(\frac{\partial v}{\partial r}\right)^2+\left(\frac{v}{r}\right)^2+\left(\frac{\partial u}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial r}\right)^2-\frac{1}{3}\left(\frac{\partial v}{\partial r}+\frac{v}{r}+\frac{\partial u}{\partial x}\right)^2\right]$$

u and v are axial and radial velocities.



a) What are the physical significance of terms marked 1 through 5 in the above equation (2.5 points)
b) Simplify the equation for a hydrodynamically and thermally fully developed flow when Pe >>1 (1.5 points)
c) For an extremely viscous fully developed flow the viscous dissipation and radial conduction terms are dominating terms and the convective term becomes negligibly small. Write the governing equation for this condition. (1 point)
d) Using parabolic velocity distribution $u=2U\left[1-\left(\frac{r}{r_o}\right)^2\right]$, where $U=\frac{r_o^2}{8\mu}\left(-\frac{dP}{dx}\right)$, derive an expression for the fully developed temperature distribution of an extremely viscous flow in the tube for a constant wall temperature of T_o (3.5 points)
e) Show that the total heat transfer rate through the pipe wall over a given length L is equal its pumping power, i.e, $Q=(\dot{m}\Delta p)/\rho$
where
Q = the total heat transfer rate through the wall pipe over a given length L
$\dot{m}$ = mass flow rate through the pipe
Δp = pressure drop over the length of pipe, L (1.5 points)

602

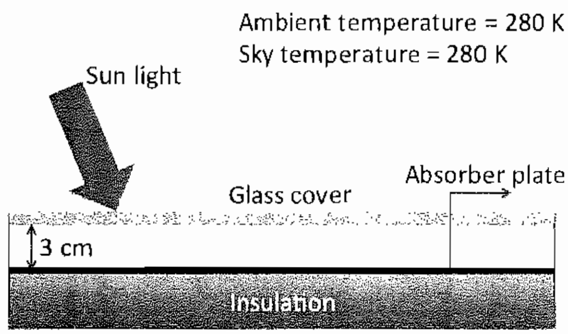
HEAT TRANSFER

Doctoral Qualifying Examination, January 2014
Mechanical Engineering, Columbia University

Problem 3: Steady state analysis of a solar flat plate collector

The schematic of a flat plate collector, which acts as a solar air heater, is shown in the figure below. The total solar flux incident on the absorber plate is 400 Wm^{-2}. For wavelengths corresponding to solar radiation, the transmissivity of the glass cover and the absorptivity of the absorber plate can be taken to be 1 and 0.95 respectively. In infrared wavelengths, the emissivity/absorptivity of the glass cover and the emissivity of the absorber plate can be taken to be 1 and 0.1 respectively. The gap between the absorber plate and the glass cover is $L = 3$ cm.

The convective heat transfer coefficient between the cover glass and the ambient is 10 $Wm^{-2}K^{-1}$. The convective heat transfer coefficient for the enclosed space between the absorber plate and the glass cover can be determined from the following correlation for Nusselt number:



$$Nu_L = \frac{hL}{k} = \begin{cases} 1 & Ra_L < 1708 \\ 1 + 1.446\left(1 - \dfrac{1708}{Ra_L}\right) & 1708 < Ra_L < 5900 \\ 0.229 Ra_L^{0.252} & 5900 < Ra_L < 9.23\times10^4 \\ 0.157 Ra_L^{0.285} & 9.23\times10^4 < Ra_L < 10^6 \end{cases} \qquad Ra_L = \frac{g\beta\Delta T L^3}{\nu\alpha}$$

Answer the following questions:

1. What are the different modes of heat transfer within the enclosed cavity and between the cover glass and the ambient? (1 pts)
2. Draw a thermal resistance diagram for the heat transfer from the absorber plate to the ambient. Any heat loss through the insulation at the bottom or side losses can be neglected. Linearize any thermal radiation transfer and calculate the values of the thermal resistances. Use the properties of air given below. To find the resistances, you can assume for a first guess that the temperature of the plate is 360 K and that of the cover glass is 300 K. (5 pts)
3. Find the temperature of the absorber plate. (4 pts)

Properties of air: $\nu = 19.06\times10^{-6}\ m^2s^{-1}$; $\alpha = 27.4\times10^{-6}\ m^2s^{-1}$; $k = 0.0291\ Wm^{-1}K^{-1}$; $g = 9.8\ ms^{-2}$; $\beta = 1/T$, where T is average temperature in K.

Doctoral Qualifying Examination, January 2013
Mechanical Engineering, Columbia University
HEAT TRANSFER

Problem 1:

For the steady incompressible laminar flow through a round tube, the energy equation is given by:

$$\frac{1}{\alpha}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}\right)=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial x^2}$$

Using appropriate scaling, determine:

a) The governing equation for hydro-dynamically fully developed region.

b) The condition under which the axial conduction in a fully developed circular tube can be neglected.

c) The scale of Nusselt number for fully developed flow in circular tubes when the axial conduction is negligible

Doctoral Qualifying Examination, January 2013
Mechanical Engineering, Columbia University
HEAT TRANSFER

Problem 2:

Determine the local Nusselt number along a flat plate wall with uniform heat flux using the integral method, for both $\Pr > 1$ and $\Pr < 1$. Assume linear velocity and temperature profiles, i.e.,

$\frac{u}{U_\infty} = \frac{y}{\delta}$ for hydrodynamic Boundary Layer and $\frac{T_w - T}{T_w - T_\infty} = \frac{y}{\delta_T}$ for the thermal Boundary Layer. You may take advantage of the fact that the flow part of the problem has already been solved. The flow solution for the boundary layer thickness is given by: $\delta = 3.46 x \mathrm{Re}_x^{-1/2}$

Doctoral Qualifying Examination, January 2013
Mechanical Engineering, Columbia University
HEAT TRANSFER

Problem 3.
Rewritable DVDs are made by embedding a 20 nm thick crystalline binary compound storage material between two 1 mm thick polycarbonate sheets (see Fig. 1 below). Data are written onto the storage material by irradiating it with a laser of power 1 mW and beam diameter 0.4 μm, resulting in rapid heating of the irradiated region. Since polycarbonate is transparent to the incident laser, all of the absorbed power is deposited in the storage material. The storage is achieved by inducing a crystalline to amorphous phase change in the storage material, which occurs at a temperature of 900 K. After the temperature of the spot has reached 900 K, the laser is turned off and the temperature is allowed to cool rapidly. The resulting amorphous spot has a reflectance/transmittance different from the crystalline material, thereby allowing a low power reading laser to subsequently "read" the data by detecting the changes in transmitted laser power. The voltage output of the detector in Fig. 1 is used to determine whether a bit is ON or OFF. You are asked to analyze the heat transfer involved in the process of writing a data bit onto a DVD. The absorptivity of the binary compound storage material is 0.8. Properties of polycarbonate: Density – 1200 kg/m^3, Thermal conductivity - 0.21 W/m/K, Specific heat – 1260 J/kg/K.
Determine the irradiation time (or write time) necessary to increase the temperature of a spot from 300 K to 1000 K. Keeping in mind that the time taken to write a bit is very small, usually less than microseconds, state the assumptions you are making to create a thermal model for the physical situation.

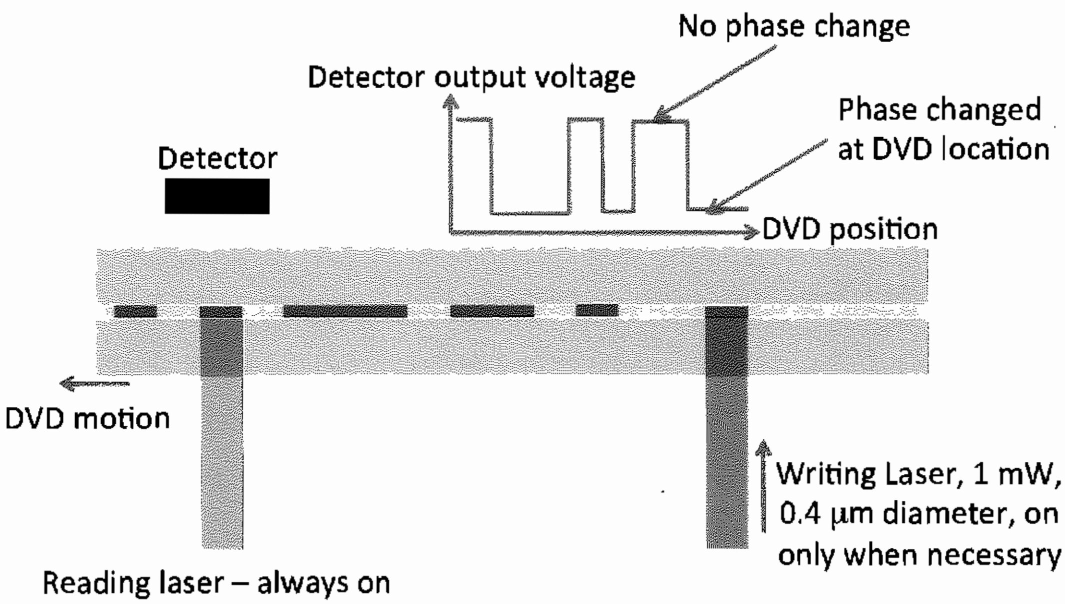


Figure 1 Schematic for laser writing of optical DVDs

LAPLACE TRANSFORM PAIRS

	$f(t)$	$\hat{f}(s)$
1.	1	$\dfrac{1}{s}$
2.	e^{at}	$\dfrac{1}{s-a}$
3.	$\sin at$	$\dfrac{a}{s^2+a^2}$
4.	$\cos at$	$\dfrac{s}{s^2+a^2}$
5.	$\sinh at$	$\dfrac{a}{s^2-a^2}$
6.	$\cosh at$	$\dfrac{s}{s^2-a^2}$
7.	$af(t)$	$a\hat{f}(s)$
8.	$f(t)+g(t)$	$\hat{f}(s)+\hat{g}(s)$
9.	$f'(t)$	$-f(0)+s\hat{f}(s)$
10.	$f''(t)$	$-f'(0)-sf(0)+s^2\hat{f}(s)$
11.	$tf(t)$	$-\dfrac{d}{ds}\hat{f}(s)$
12.	$\int_{\sigma=0}^{t} f(\sigma)g(t-\sigma)\,d\sigma$	$\hat{f}(s)\hat{g}(s)$

	$f(t)$	$\hat{f}(s)$
13.	$\sum_{k=1}^{N} \frac{P(s_k)}{Q'(s_k)} e^{s_k t}$ where the s_k are the roots of $Q(s) = 0$	$\frac{P(s)}{Q(s)}$ where $P(s)$ and $Q(s)$ are polynomials in s
14.	$1(t - a)$ where $1(\alpha) = \begin{cases} 0 & \alpha < 0 \\ 1 & \alpha \geq 0 \end{cases}$	$\frac{e^{-as}}{s}$
15.	$g(t - a)1(t - a)$	$\hat{g}(s)e^{-as}$
16.	$1(t - a)(1 - e^{-b(t-a)})$	$\frac{be^{-as}}{s(s + b)}$
17.	$t^{\nu/2} J_\nu(a\sqrt{t})$	$\frac{a^\nu}{2^\nu s^{\nu+1}} e^{-a^2/4s}$
18.	$\frac{1}{\sqrt{at}} e^{-bt} I_1(2\sqrt{t/a})$	$e^{1/[a(s+b)]} - 1$
19.	$e^{-bt} I_0(2\sqrt{at})$	$\frac{e^{a/(s+b)}}{s + b}$
20.	$J_0(at)$	$\frac{1}{\sqrt{s^2 + a^2}}$
21.	$J_n(at)$	$\frac{a^n}{\sqrt{s^2 + a^2}(\sqrt{s^2 + a^2} + s)^n}$ where n = nonnegative integer
22.	$\frac{a}{2\sqrt{\pi t^3}} e^{-(a^2/4t) - bt}$	$\exp(-a\sqrt{s + b})$
23.	$\frac{1}{\sqrt{\pi t}} e^{-a^2/4t}$	$\frac{\exp(-a\sqrt{s})}{\sqrt{s}}$
24.	$\operatorname{erfc} \frac{a}{2\sqrt{t}}$	$\frac{\exp(-a\sqrt{s})}{s}$
25.	$2\sqrt{\frac{t}{\pi}} e^{-(a^2/4t)} - a \operatorname{erfc} \frac{a}{2\sqrt{t}}$	$s^{-3/2} \exp(-a\sqrt{s})$
26.	$e^{ab} e^{b^2 t} \operatorname{erfc}\left(b\sqrt{t} + \frac{a}{2\sqrt{t}}\right)$	$\frac{\exp(-a\sqrt{s})}{\sqrt{s}(b + \sqrt{s})}$

	$f(t)$	$\hat{f}(s)$
27.	$\operatorname{erfc}\left(\dfrac{a}{2\sqrt{t}}\right) - e^{b+(b^2t/a^2)}\operatorname{erfc}\left(\dfrac{a}{2\sqrt{t}} + \dfrac{b\sqrt{t}}{a}\right)$	$\dfrac{b\exp(-a\sqrt{s})}{s(b + a\sqrt{s})}$
28.	$-\dfrac{2\pi}{L^2}\sum_{n=1}^{\infty}(-1)^n n \sin\dfrac{n\pi x}{L}\, e^{-(n^2\pi^2 t)/L^2}$	$\dfrac{\sinh x\sqrt{s}}{\sinh L\sqrt{s}}$ $(-L < x < +L)$
29.	$1 + 2\sum_{n=1}^{\infty}\cos 2n\pi x e^{-n^2\pi^2 t}$	$\dfrac{\cosh[(2x-1)\sqrt{s}]}{\sqrt{s}\sinh\sqrt{s}}$ $(0 \le x \le 1)$

ADVANCED HEAT TRANSFER

WATER AT ATMOSPHERIC PRESSURE

T (°C)	ρ (g/cm^3)	c_p (kJ/kg · K)	c_v (kJ/kg · K)	h_{fg} (kJ/kg)	β (K^{-1})
0	0.9999	4.217	4.215	2501	-0.6×10^{-4}
5	1	4.202	4.202	2489	$+0.1 \times 10^{-4}$
10	0.9997	4.192	4.187	2477	0.9×10^{-4}
15	0.9991	4.186	4.173	2465	1.5×10^{-4}
20	0.9982	4.182	4.158	2454	2.1×10^{-4}
25	0.9971	4.179	4.138	2442	2.6×10^{-4}
30	0.9957	4.178	4.118	2430	3.0×10^{-4}
35	0.9941	4.178	4.108	2418	3.4×10^{-4}
40	0.9923	4.178	4.088	2406	3.8×10^{-4}
50	0.9881	4.180	4.050	2382	4.5×10^{-4}
60	0.9832	4.184	4.004	2357	5.1×10^{-4}
70	0.9778	4.189	3.959	2333	5.7×10^{-4}
80	0.9718	4.196	3.906	2308	6.2×10^{-4}
90	0.9653	4.205	3.865	2283	6.7×10^{-4}
100[a]	0.9584	4.216	3.816	2257	7.1×10^{-4}

T (°C)	μ (g/cm · s)	ν (cm^2/s)	k (W/m · K)	α (cm^2/s)	Pr	$\frac{g\beta}{\alpha\nu} = \frac{Ra_H}{H^3\,\Delta T}$ (K^{-1} · cm^{-3})
0	0.01787	0.01787	0.56	0.00133	13.44	-2.48×10^3
5	0.01514	0.01514	0.57	0.00136	11.13	$+0.47 \times 10^3$
10	0.01304	0.01304	0.58	0.00138	9.45	4.91×10^3
15	0.01137	0.01138	0.59	0.00140	8.13	9.24×10^3
20	0.01002	0.01004	0.59	0.00142	7.07	14.45×10^3
25	0.00891	0.00894	0.60	0.00144	6.21	19.81×10^3
30	0.00798	0.00802	0.61	0.00146	5.49	25.13×10^3
35	0.00720	0.00725	0.62	0.00149	4.87	30.88×10^3
40	0.00654	0.00659	0.63	0.00152	4.34	37.21×10^3
50	0.00548	0.00554	0.64	0.00155	3.57	51.41×10^3
60	0.00467	0.00475	0.65	0.00158	3.01	66.66×10^3
70	0.00405	0.00414	0.66	0.00161	2.57	83.89×10^3
80	0.00355	0.00366	0.67	0.00164	2.23	101.3×10^3
90	0.00316	0.00327	0.67	0.00165	1.98	121.8×10^3
100	0.00283	0.00295	0.68	0.00166	1.78	142.2×10^3

Source: Data collected from Refs. 1–3.

[a]Saturated.

DRY AIR AT ATMOSPHERIC PRESSURE

T (°C)	ρ (kg/m³)	c_P (kJ/kg · K)	μ (kg/s · m)	ν (cm²/s)	k (W/m · K)	α (cm²/s)	Pr	$\frac{g\beta}{\alpha\nu} = \frac{Ra_H}{H^3 \Delta T}$ (cm⁻³ · K⁻¹)
−180	3.72	1.035	6.50×10^{-6}	0.0175	0.0076	0.019	0.92	3.2×10^4
−100	2.04	1.010	1.16×10^{-5}	0.057	0.016	0.076	0.75	1.3×10^3
−50	1.582	1.006	1.45×10^{-5}	0.092	0.020	0.130	0.72	367
0	1.293	1.006	1.71×10^{-5}	0.132	0.024	0.184	0.72	148
10	1.247	1.006	1.76×10^{-5}	0.141	0.025	0.196	0.72	125
20	1.205	1.006	1.81×10^{-5}	0.150	0.025	0.208	0.72	107
30	1.165	1.006	1.86×10^{-5}	0.160	0.026	0.223	0.72	90.7
60	1.060	1.008	2.00×10^{-5}	0.188	0.028	0.274	0.70	57.1
100	0.946	1.011	2.18×10^{-5}	0.230	0.032	0.328	0.70	34.8
200	0.746	1.025	2.58×10^{-5}	0.346	0.039	0.519	0.68	9.53
300	0.616	1.045	2.95×10^{-5}	0.481	0.045	0.717	0.68	4.96
500	0.456	1.093	3.58×10^{-5}	0.785	0.056	1.140	0.70	1.42
1000	0.277	1.185	4.82×10^{-5}	1.745	0.076	2.424	0.72	0.18

Source: Data collected from Refs. 1–3.

ADVANCED HEAT TRANSFER

Doctoral Qualifying Examination, January 2012

Mechanical Engineering, Columbia University

Problem 1:

Write the Navier-Stoke and energy equations for a two-dimensional incompressible flow with constant properties (2 points). Using appropriate scaling (order of magnitude analysis) reduce those equations to the boundary layer equations (2 points). Then come up with scales of the hydrodynamic boundary layer thickness (δ), thermal boundary layer thickness (δ_T) and Nu number for $Pr >> 1$ and $Pr << 1$ (6 points).

ADVANCED HEAT TRANSFER

Doctoral Qualifying Examination, January 2012

Mechanical Engineering, Columbia University

Problem 2:

The airflow through the gaps formed at the top and bottom of a closed door is driven by the local air pressure difference between the two sides of the door (see the figure). The door separates two isothermal rooms at different temperatures, T_c and T_h. In each room, the pressure distribution is purely hydrostatic, $P_c(y)$ and $P_h(y)$, and the height-averaged pressure is the same on both sides of the door on both sides of the door, i.e., $P_h(H/2) = P_c(H/2)$.

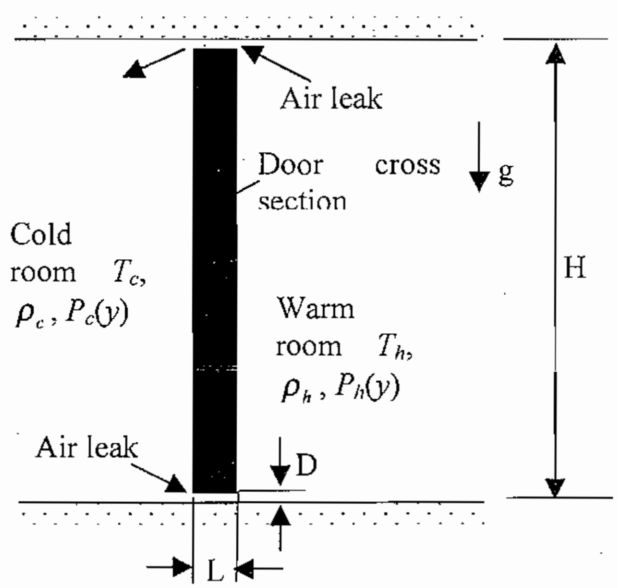
(a) Assume that the airflow through each gap is laminar and fully developed. Derive an expression for air mass flow rate ($\dot{m}$) in terms of hot and cold room densities (ρ_h and ρ_c), gap opening and length (D, L), door height and width and height (W, H), ν and g. (4 points)

(b) Derive an expression for the net convection heat transfer from the warm room to the cold room , through the two gaps in terms of dimensions shown in the figure and fluid properties. (4 points)

(c) If T_c=10°C, T_h=30°C, D=0.5 mm, L=5 cm, H=2.2 m, and W=1.5 m. Calculate the mass flow rate and convection heat transfer through the two gaps. Comment on how these quantities react to an increase in the gap thickness. (2 points)

Note: the average fluid velocity for a fully developed laminar flow through a gap of height D, length L is:

$$V_{avg} = \frac{D^2}{12\mu}\frac{\Delta P}{L}$$



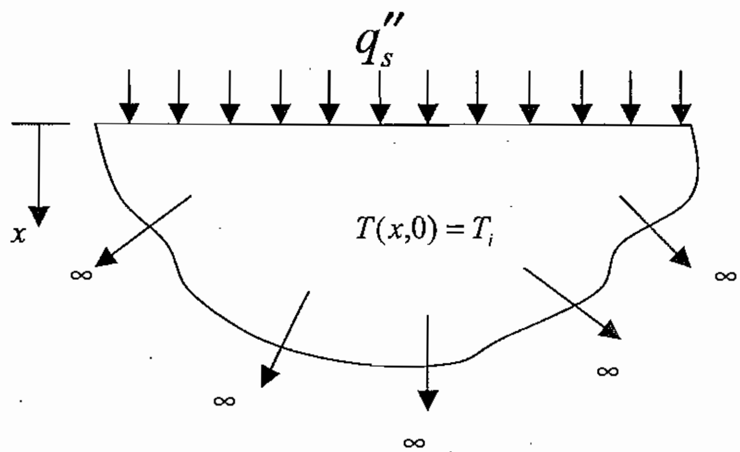
525

ADVANCED HEAT TRANSFER

Doctoral Qualifying Examination, January 2012
Mechanical Engineering, Columbia University

Problem 3:

1. A semi-infinite media shown below initially at temperature of T_i is suddenly ($t > 0$) exposed to a surface heat flux q_s''. Determine the transient temperature distribution, $T(x,t)$. (10 points)



ADVANCED HEAT TRANSFER

Doctoral Qualifying Examination, January 2011

Mechanical Engineering, Columbia University

Problem 1:

The energy equation for a laminar, incompressible flow with constant properties in a circular tube is given by:

$$\frac{1}{\alpha}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}\right)=\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial x^2}$$

Determine:

a. The condition under which the axial conduction in a fully developed flow in a circular tube can be neglected. (4 points)

b. The scale of Nusselt number for fully developed flow in circular tubes when the axial conduction is negligible. Use U as average axial velocity and D as tube diameter. (6 points)

ADVANCED HEAT TRANSFER

Doctoral Qualifying Examination, January 2011

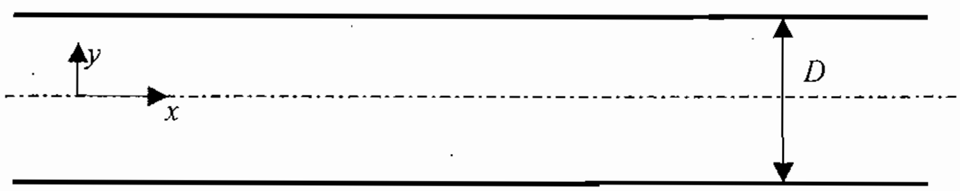
Mechanical Engineering, Columbia University

Problem 2:

Consider fully developed laminar flow in a channel formed by two parallel plates. The distance between the two plates is D, as shown in the following figure. The heat flux through both walls of the channel is uniform, q''. The velocity profile between the two plates is given by:

$$u(y)=\frac{3}{2}U\left[1-\left(\frac{y}{D/2}\right)^2\right]=\frac{3}{2}U\left[1-\eta^2\right] \qquad \text{where} \quad \eta=\frac{y}{D/2}$$

and U is the average fluid velocity



For the specified condition:

a. Write the governing differential equation for the fully developed temperature profile. (2 points)
b. Determine the temperature gradient in x-direction, i.e. $\partial T/\partial x$ and $\partial T_m/\partial x$. (2 points)
c. Define an appropriate dimensionless temperature that is only function of y-coordinate. (2 points)
d. Derive an analytical expression for the fully developed dimensionless temperature profile. (2 points)
e. Determine the value of Nusselt number based on distance between the two plates, i.e. $\mathrm{Nu}=\frac{hD}{k}$. (2 points)

ADVANCED HEAT TRANSFER

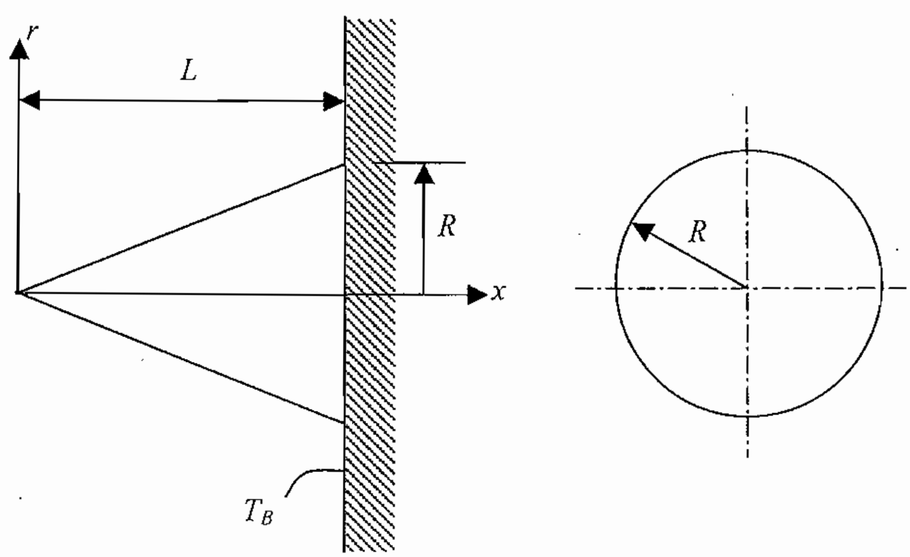
Doctoral Qualifying Examination, January 2011

Mechanical Engineering, Columbia University

Problem 3:

A spine protruding from a wall at temperature T_B has a shape of a circular cone. The radius of the cone base is R. The spine comes to a point at its tip and its length is L, as shown in the following figure. Assume that the temperature is uniform in the r direction.

a. Derive the governing ordinary differential equation from basic principles (3 points)
b. Obtain the solution to this equation (7 points)



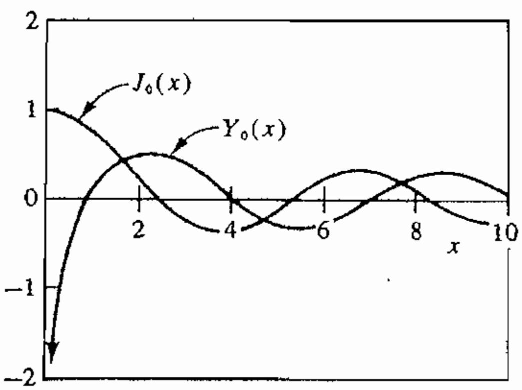
The following information may be helpful.

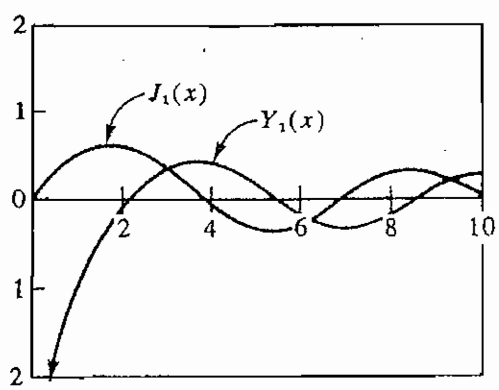
General Bessel Equation

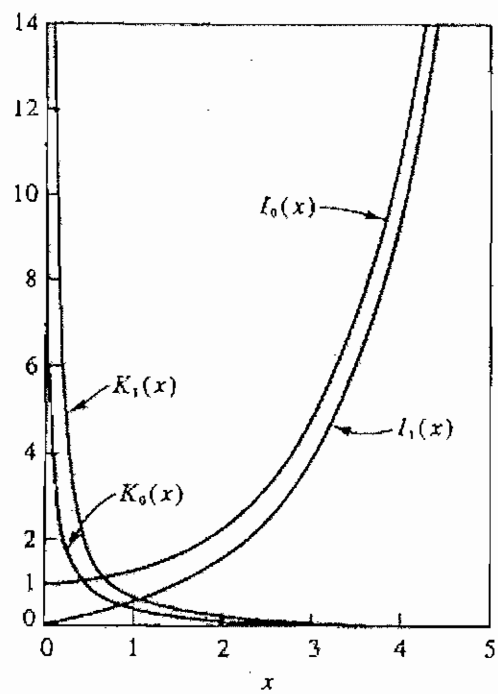
$$x^2 u'' + [(1 - 2A)x - 2Bx^2]u' + [C^2 D^2 x^{2C} + B^2 x^2 - B(1 - 2A)x + A^2 - C^2 n^2]u = 0 \tag{2·4·1}$$

This has the solution

$$u = x^A e^{Bx}[C_1 J_n(Dx^C) + C_2 Y_n(Dx^C)] \tag{2·4·2}$$







ADVANCED HEAT TRANSFER

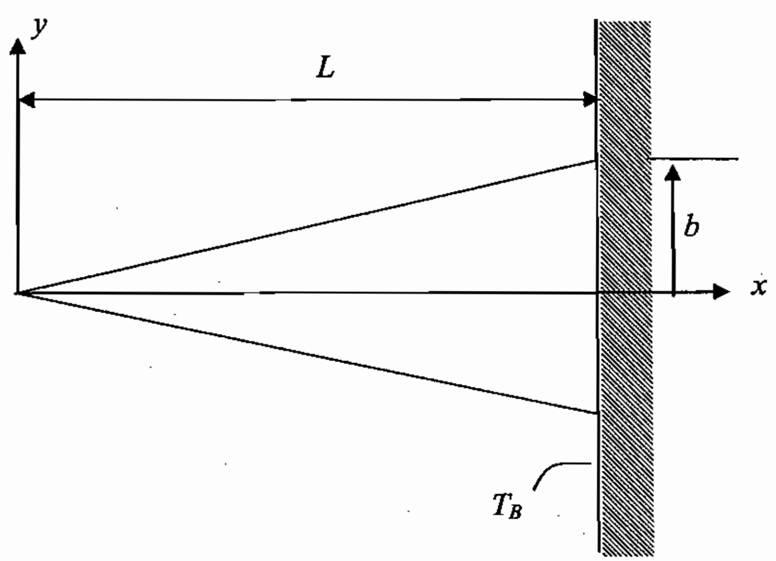
Doctoral Qualifying Examination, January 2010
Mechanical Engineering, Columbia University

Problem 1:

A straight triangular fin shown in the figure has a base at temperature T_B. The thickness of the fin at its base (x=L) is $2b$, and at its tip (x=0) is zero. The fin's length is L.

- **a)** Derive the governing equation for the temperature distribution in the fin from the basic principles
- **b)** Obtain an expression for the temperature distribution in the fin

Assume that the temperature is uniform in the y direction.



HEAT TRANSFER

Doctoral Qualifying Examination, January 2010
Mechanical Engineering, Columbia University

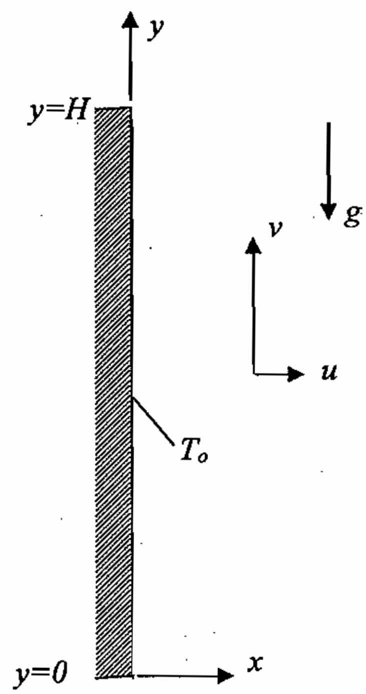
Problem 2:

The governing equations for natural convection of a laminar boundary layer over a vertical surface are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}=0$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\nu\frac{\partial^2 v}{\partial x^2}+g\beta\left(T-T_\infty\right)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial x^2}$$



a) Write pertaining boundary conditions
b) Determine scale of Nusselt numbers for both high and low Prandtl number fluids

Problem 3:
A slug flow (uniform velocity profile U) is heated by passing between two parallel plates. The spacing between the plates is D and they are isothermal at T_w. The flow is thermally fully developed. Determine:

a) An analytical expression for the fully developed temperature profile and the longitudinal variation of mean temperature.

b) Show that the Nusselt number based on the hydraulic diameter is equal to π^2.

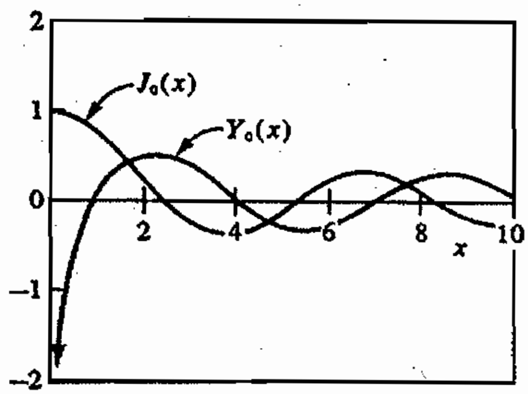
The following information may be helpful.

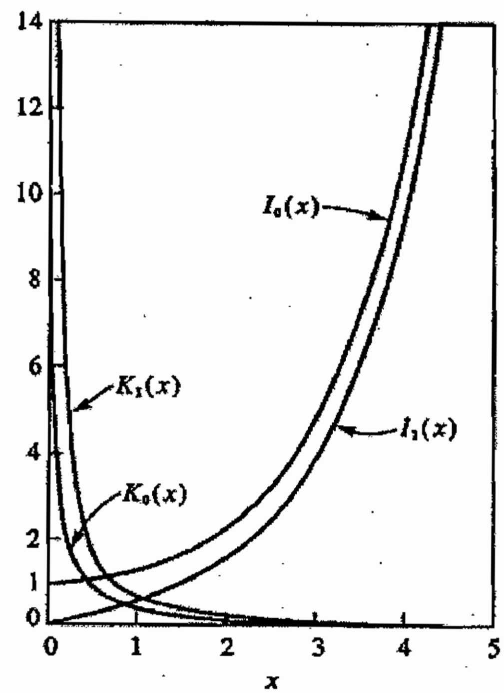
General Bessel Equation

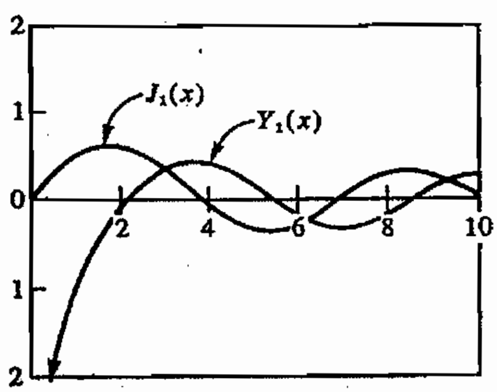
$$x^2u'' + [(1 - 2A)x - 2Bx^2]u' + [C^2D^2x^{2C} + B^2x^2 - B(1 - 2A)x + A^2 - C^2n^2]u = 0 \tag{2·4·1}$$

This has the solution

$$u = x^A e^{Bx}[C_1 J_n(Dx^C) + C_2 Y_n(Dx^C)] \tag{2·4·2}$$







424

HEAT TRANSFER

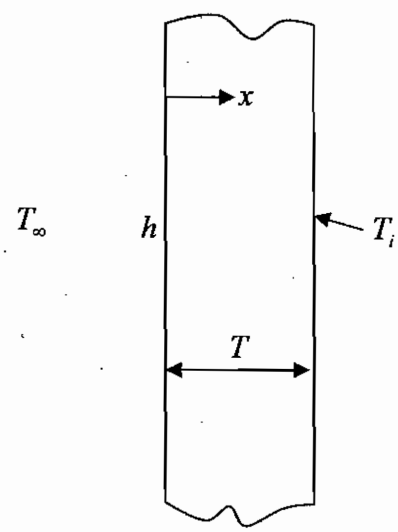
Problem 1:

A plane wall with thickness of L and thermal diffusivity of α initially is at a uniform temperature T_i. The ambient temperature on one side (left side) of the wall is suddenly changed to a new value T_∞, while the temperature on the other side (right side) of the wall is held at its initial value, as shown in the figure. The heat transfer coefficient between the surface and ambient is h.

a) Write the governing equation, boundary and initial conditions

b) Determine the temperature distribution as a function of time and position x

c) Determine heat transferred from the ambient to the wall as a function of time.



425

HEAT TRANSFER

Problem 2:

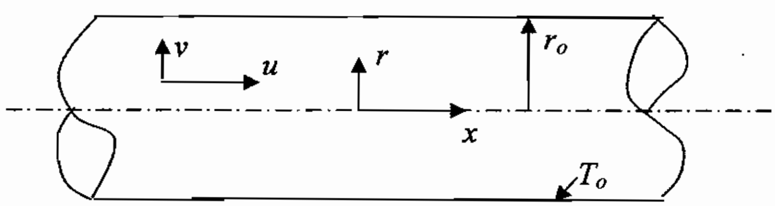
The energy equation for an axisymmetric and incompressible laminar flow in a circular tube is given by:

$$\rho c_p\left(\overbrace{u\frac{\partial T}{\partial x}}^{1}+\overbrace{v\frac{\partial T}{\partial r}}^{2}\right)=k\left[\overbrace{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)}^{3}+\overbrace{\frac{\partial^2 T}{\partial x^2}}^{4}\right]+\overbrace{\mu\phi}^{5}$$

where

$$\phi=2\left[\left(\frac{\partial v}{\partial r}\right)^2+\left(\frac{v}{r}\right)^2+\left(\frac{\partial u}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial r}\right)^2-\frac{1}{3}\left(\frac{\partial v}{\partial r}+\frac{v}{r}+\frac{\partial u}{\partial x}\right)^2\right]$$

u and v are axial and radial velocities.



a) What are the physical significance of terms marked 1 through 5 in the above equation
b) Simplify the equation for a hydrodynamically and thermally fully developed flow when Pe >>1
c) For an extremely viscous fully developed flow the viscous dissipation and radial conduction terms are dominating terms and the convective term becomes negligibly small. Write the governing equation for this condition.
d) Using parabolic velocity distribution $u=2U\left[1-\left(\frac{r}{r_o}\right)^2\right]$, where $U=\frac{r_o^2}{8\mu}\left(-\frac{dP}{dx}\right)$, derive an expression for temperature distribution of an extremely viscous flow in the tube for a constant wall temperature of T_o
e) Show that the total heat transfer rate through the pipe wall over a given length L is equal its pumping power, i.e, $Q=(\dot{m}\Delta p)/\rho$
   where
   Q = the total heat transfer rate through the wall pipe over a given length L
   $\dot{m}$ = mass flow rate through the pipe
   Δp = pressure drop over the length of pipe, L.

426

HEAT TRANSFER

Problem 3:

Vapor degreasing is a method of cleaning surfaces to remove grease and oil remnants on the surface. A schematic of a vapor degreasing is shown in the figure at the bottom of this page. It consists of a tank with the cleaning solvent at the bottom, which can be heated by the heater. When the heater is hot enough, the chamber is filled with the vapor at the saturation temperature. The plate that is to be degreased is at a lower temperature and the vapors condense on either side of the plate. As the liquid falls on the plate it removes the grease and oil stains that are on the plate. The advantage of this method is that you always have pure vapor doing the cleaning. This cleaning action continues until the plate reaches the temperature of the vapor. An owner of one such vapor degreaser has a few questions to answer.

The plate to be cleaned is a copper plate is of length (L) = 20 cm, width (W) = 20 cm, and thickness (δ) = 0.5 mm. The cleaning agent being used is iso-propyl alcohol (IPA). The IPA vapor in the degreaser is at the saturation temperature, T_{sat} = 355.4 K (at atm. pressure). The latent heat of condensation of IPA is h_{fg} = 663.1 kJ/kg. The copper plate is placed in the degreaser at time t = 0 at temperature T_o = 290 K. You can assume that there are no spatial temperature variations in the plate.

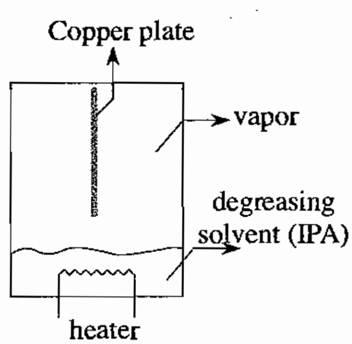
Properties of copper:
Density, $\rho = 8954\ \text{kgm}^{-3}$; specific heat, $C_p = 384\ \text{Jkg}^{-1}\ \text{K}^{-1}$

Properties of IPA, liquid:
Density $\rho = 785\ \text{kgm}^{-3}$; thermal conductivity, $k = 0.14\ \text{Wm}^{-1}\ \text{K}^{-1}$;
Viscosity $\mu = 2.43 \times 10^{-3}\ \text{Nm}^{-2}\text{s}$;
Latent heat of condensation, h_{fg} = 663.1 kJ/kg

You have to answer the following questions:

a) The owner wants to know how much solvent will be needed to clean each plate. How much vapor condenses on the copper plate as the temperature is increased from T_o = 290 K to T_{sat} = 355.4 K?

b) The owner also wants to know how long the plate should be in the vapor degreaser. Write down the differential equation for the temperature of the copper plate. Using that, determine the time taken for the plate to reach T_{sat} = 355.4 K from T= 290 K?



APPENDIX **E**

LAPLACE TRANSFORM PAIRS

	$f(t)$	$\hat{f}(s)$
1.	1	$\dfrac{1}{s}$
2.	e^{at}	$\dfrac{1}{s-a}$
3.	$\sin at$	$\dfrac{a}{s^2+a^2}$
4.	$\cos at$	$\dfrac{s}{s^2+a^2}$
5.	$\sinh at$	$\dfrac{a}{s^2-a^2}$
6.	$\cosh at$	$\dfrac{s}{s^2-a^2}$
7.	$af(t)$	$a\hat{f}(s)$
8.	$f(t)+g(t)$	$\hat{f}(s)+\hat{g}(s)$
9.	$f'(t)$	$-f(0)+s\hat{f}(s)$
10.	$f''(t)$	$-f'(0)-sf(0)+s^2\hat{f}(s)$
11.	$tf(t)$	$-\dfrac{d}{ds}\hat{f}(s)$
12.	$\displaystyle\int_{\sigma=0}^{t} f(\sigma)g(t-\sigma)\,d\sigma$	$\hat{f}(s)\hat{g}(s)$

436

$f(t)$	$\hat{f}(s)$
13. $\displaystyle\sum_{k=1}^{N} \frac{P(s_k)}{Q'(s_k)} e^{s_k t}$ where the s_k are the roots of $Q(s) = 0$	$\dfrac{P(s)}{Q(s)}$ where $P(s)$ and $Q(s)$ are polynomials in s
14. $1(t - a)$ where $1(\alpha) = \begin{cases} 0 & \alpha < 0 \\ 1 & \alpha \geq 0 \end{cases}$	$\dfrac{e^{-as}}{s}$
15. $g(t - a)1(t - a)$	$\hat{g}(s)e^{-as}$
16. $1(t - a)(1 - e^{-b(t-a)})$	$\dfrac{be^{-as}}{s(s + b)}$
17. $t^{\nu/2} J_\nu(a\sqrt{t})$	$\dfrac{a^\nu}{2^\nu s^{\nu+1}} e^{-a^2/4s}$
18. $\dfrac{1}{\sqrt{at}} e^{-bt} I_1(2\sqrt{t/a})$	$e^{1/[a(s+b)]} - 1$
19. $e^{-bt} I_0(2\sqrt{at})$	$\dfrac{e^{a/(s+b)}}{s + b}$
20. $J_0(at)$	$\dfrac{1}{\sqrt{s^2 + a^2}}$
21. $J_n(at)$	$\dfrac{a^n}{\sqrt{s^2 + a^2}(\sqrt{s^2 + a^2} + s)^n}$ where n = nonnegative integer
22. $\dfrac{a}{2\sqrt{\pi t^3}} e^{-(a^2/4t) - bt}$	$\exp(-a\sqrt{s + b})$
23. $\dfrac{1}{\sqrt{\pi t}} e^{-a^2/4t}$	$\dfrac{\exp(-a\sqrt{s})}{\sqrt{s}}$
24. $\operatorname{erfc} \dfrac{a}{2\sqrt{t}}$	$\dfrac{\exp(-a\sqrt{s})}{s}$
25. $2\sqrt{\dfrac{t}{\pi}} e^{-(a^2/4t)} - a \operatorname{erfc} \dfrac{a}{2\sqrt{t}}$	$s^{-\frac{3}{2}} \exp(-a\sqrt{s})$
26. $e^{ab}e^{b^2 t} \operatorname{erfc}\left(b\sqrt{t} + \dfrac{a}{2\sqrt{t}}\right)$	$\dfrac{\exp(-a\sqrt{s})}{\sqrt{s}(b + \sqrt{s})}$

	$f(t)$	$\hat{f}(s)$
27.	$\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right) - e^{b+(b^2t/a^2)}\operatorname{erfc}\left(\frac{a}{2\sqrt{t}} + \frac{b\sqrt{t}}{a}\right)$	$\frac{b\exp(-a\sqrt{s})}{s(b + a\sqrt{s})}$
28.	$-\frac{2\pi}{L^2}\sum_{n=1}^{\infty}(-1)^n n \sin\frac{n\pi x}{L}e^{-(n^2\pi^2t)/L^2}$	$\frac{\sinh x\sqrt{s}}{\sinh L\sqrt{s}}$ $(-L < x < +L)$
29.	$1 + 2\sum_{n=1}^{\infty}\cos 2n\pi x e^{-n^2\pi^2t}$	$\frac{\cosh[(2x - 1)\sqrt{s}]}{\sqrt{s}\sinh\sqrt{s}}$ $(0 \le x \le 1)$

HEAT TRANSFER

Doctoral Qualifying Examination, January 2008
Mechanical Engineering, Columbia University

Problem 1:

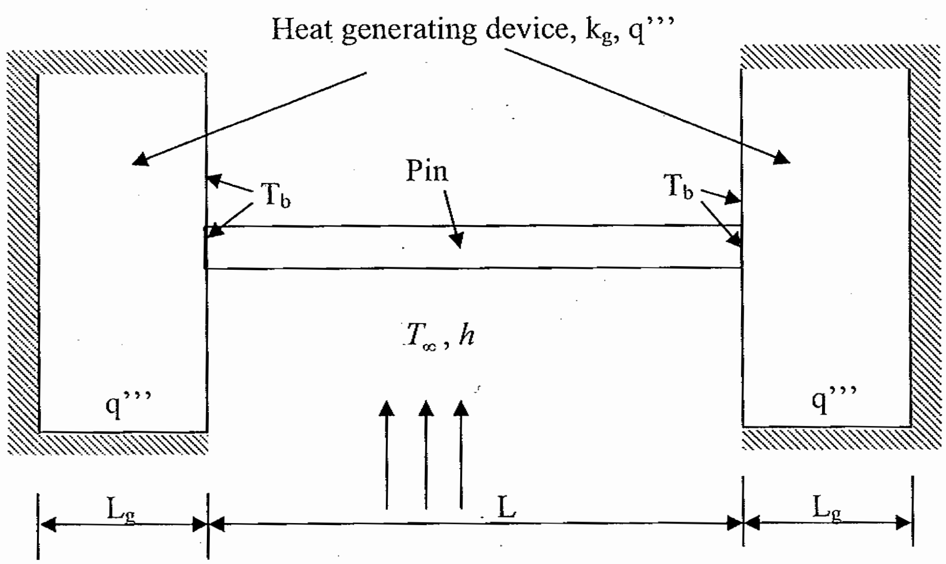
Write the two dimensional Navier-Stoke and energy equations for an incompressible flow with constant properties. Using appropriate scaling (order of magnitude analysis) reduce those equations to the boundary layer equations. Then come up with scales of the hydrodynamic boundary layer thickness (δ), thermal boundary layer thickness (δ_T) and Nu number for $Pr >> 1$ and $Pr << 1$.

HEAT TRANSFER

Doctoral Qualifying Examination, January 2008
Mechanical Engineering, Columbia University

Problem 2:

Pin fins are widely used in electronic systems to provide cooling as well as to support devices. Consider the pin of uniform diameter D, length L, and thermal conductivity of k connecting two identical devices of length L_g and surface area of A_g. The devices are characterized by a uniform volumetric generation of thermal energy q''' and thermal conductivity of k_g. Assume that the exposed surfaces of the devices are at a uniform temperature corresponding to that of the pin base, T_b, and that heat is transferred by convection from the exposed surfaces to an adjoining fluid. The back sides of the devices are perfectly insulated. Derive an expression for the base temperature T_b in terms of the device parameters (k_g, q''', L_g, A_g) and convection parameters (T_∞, h), and the pin parameters (k, D, L).

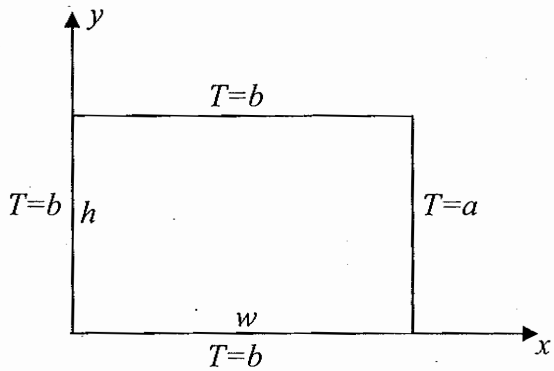


HEAT TRANSFER

Doctoral Qualifying Examination, January 2008
Mechanical Engineering, Columbia University

Problem 3:

Determine the two-dimensional temperature distribution in the following slab.



HEAT TRANSFER

Doctoral Qualifying Examination, January 2007
Mechanical Engineering, Columbia University

Problem 1:

For the steady incompressible laminar flow through a round tube, the energy equation is given by:

$$\frac{1}{\alpha}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}\right)=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial x^2}$$

Using appropriate scaling, determine:

a) The governing equation for hydrodynamically fully developed region.

b) The condition under which the axial conduction in a fully developed circular tube can be neglected.

c) The scale of Nusselt number for fully developed flow in circular tubes when the axial conduction is negligible

HEAT TRANSFER

Doctoral Qualifying Examination, January 2007
Mechanical Engineering, Columbia University

Problem 2:

Determine the local Nusselt number along a flat plate wall with uniform heat flux using the integral method, for both $\Pr > 1$ and $\Pr < 1$. Assume linear velocity and temperature profiles, i.e.,

$\frac{u}{U_\infty} = \frac{y}{\delta}$ for hydrodynamic Boundary Layer and $\frac{T_w - T}{T_w - T_\infty} = \frac{y}{\delta_T}$ for the thermal Boundary Layer. You may take advantage of the fact that the flow part of the problem has already been solved. The flow solution for the boundary layer thickness is given by: $\delta = 3.46x\,\mathrm{Re}_x^{-1/2}$

HEAT TRANSFER

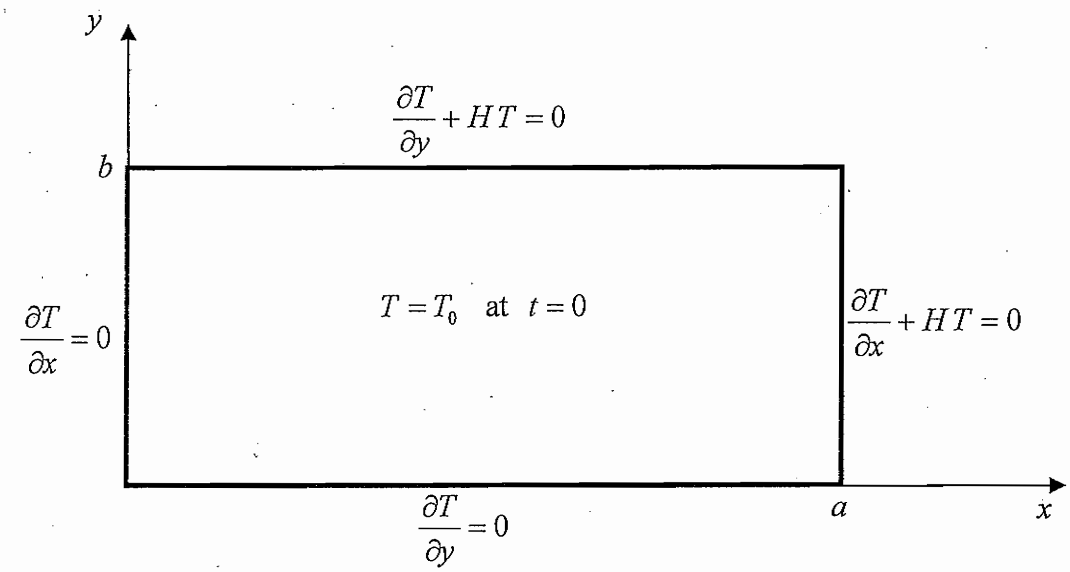
Doctoral Qualifying Examination, January 2007
Mechanical Engineering, Columbia University

Problem 3:

Obtain a transient temperature distribution for a slab ($0 \le x \le L$) initially at a temperature of T_0, when the boundary condition are: insulated at $x = 0$ and transfers heat via convection at $x = L$, i.e.,

$$\frac{\partial T(x,t)}{\partial x} = 0 \quad \text{at} \quad x = 0 \qquad \text{and} \qquad \frac{\partial T(x,t)}{\partial x} + HT(x,t) = 0 \quad \text{at} \quad x = L$$

Using the one dimensional transient solution obtain a solution for transient temperature distribution for the rectangular region shown in the following figure.

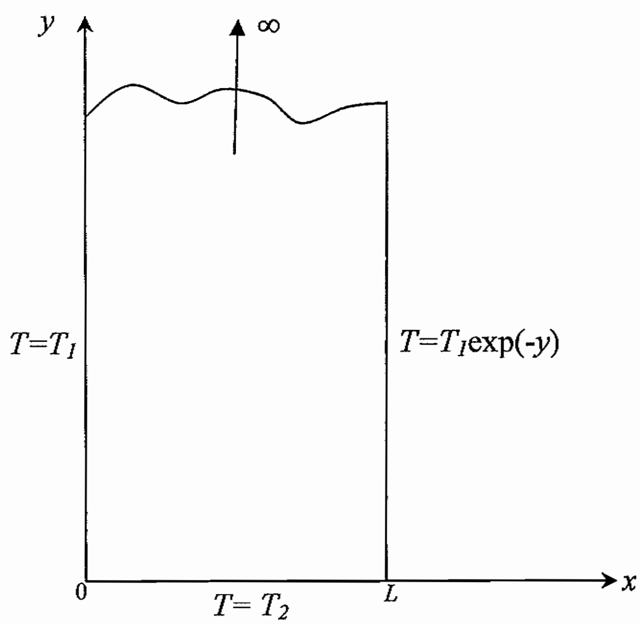


HEAT TRANSFER

Doctoral Qualifying Examination, January 2006
Mechanical Engineering, Columbia University

Problem 1:

Express algebraically the steady-state temperature distribution in the semi-infinite plate shown. The surfaces are all at constant temperatures (with respect to time), and the plate has a width of L.



HEAT TRANSFER

Doctoral Qualifying Examination, January 2006
Mechanical Engineering, Columbia University

Problem 2:

Write the two dimensional Navier-Stokes and energy equations for an incompressible flow with constant properties. Using appropriate scaling (order of magnitude analysis) reduce those equations to the boundary layer equations. Then come up with scales of the hydrodynamic boundary layer thickness (δ), thermal boundary layer thickness (δ_T) and Nu number for $Pr >> 1$ and $Pr << 1$.

HEAT TRANSFER

Doctoral Qualifying Examination, January 2006
Mechanical Engineering, Columbia University

Problem 3:

An electrical wire of diameter 1 mm is suspended horizontally in air of temperature 20°C. The Joule heating of the wire results in the heat generation at the rate of q'=0.01 W/cm. The wire can be modeled as a cylinder with isothermal surface. The ambient air is motionless. Calculate the wire temperature. For Natural Convection from a horizontal cylinder use the following correlation:

$$\mathbf{Nu}_D = \left\{0.6 + \frac{0.367\mathbf{Ra}_D^{1/6}}{\left[1+(0.559/\mathrm{Pr})^{9/16}\right]^{8/27}}\right\}^2, \text{ where } Ra_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu\alpha}$$

(At T = 27°C, air has the following properties: Pr = 0.707; $\beta = 1/T = 3.33\times10^{-3}\ K^{-1}$; $\nu = 1.59\times10^{-5}$ m^2/s; $\alpha = 2.25\times10^{-5}$ m^2/s; $k = 2.63\times10^{-2}$ W/mK .)

308

Heat Transfer

Doctoral Qualifying Examination, January 2005
Mechanical Engineering, Columbia University

Problem 1 A semi-infinite medium, $0 \le x < \infty$, is initially at zero temperature. For times $t > 0$, the boundary surface at $x = 0$ is subjected to convection with an environment at temperature T_∞ and heat transfer coefficient h. Obtain an expression for the temperature distribution $T(x,t)$ in the solid for times $t > 0$.

Problem 2 Consider the laminar boundary layer formed by the flow of 10°C water over a 10°C flat wall of length L. Show that the total shear force experienced by the wall and the mechanical power P spent on dragging the wall through the fluid is proportional to $\nu^{1/2}$.

(a) The dissipated drag power described above refers to the case in which the wall is cold as the free-stream water. Show that if the wall is heated isothermally so that its temperature rises to 90°C, the dissipated power decreases by 35 percent. In other words, show that P_h/P_c=0.65, where h and c refer to the hot- and cold-wall conditions.

(b) Compare the power savings due to heating the wall (P_c-P_h) with the electrical power needed to heat the wall to 90°C. How fast must the water flow be so that the savings in the fluid-friction power dissipation become greater than the electrical power invested in heating the wall? How short must the swept length L be so that the boundary layer remains laminar while the power savings P_c-P_h exceed the heat input to the wall?

Note: for laminar boundary layer local skin friction coefficient and the overall Nusselt number are given by the following correlations

$$C_{f,x} = 0.664\,\mathrm{Re}_x^{-1/2} \quad \text{and} \quad Nu_{0-x} = 0.664\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_x^{1/2}$$

Properties of water are given in the attached table

Problem 3 The airflow through the gaps formed at the top and bottom of a closed door is driven by the local air pressure difference between the two sides of the door (see the figure). The door separates two isothermal rooms at different temperatures, T_c and T_h. In each room, the pressure distribution is purely hydrostatic, $P_c(y)$ and $P_h(y)$, and the height-averaged pressure is the same on both sides of the door on both sides of the door, i.e., $P_h(H/2) = P_c(H/2)$.

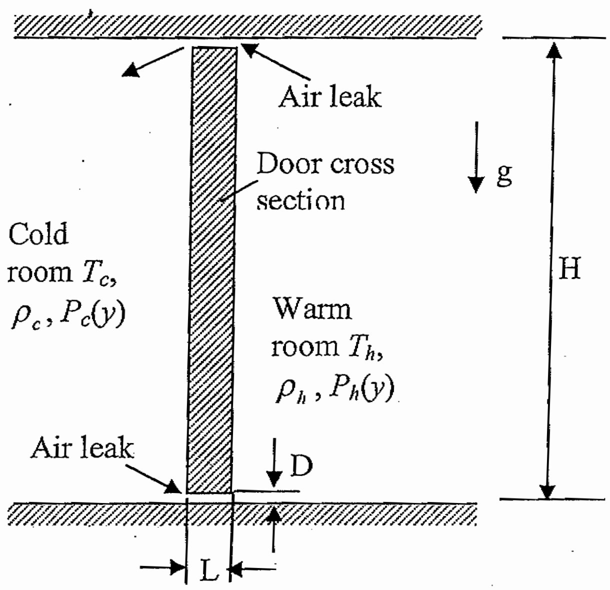
(a) Assume that the airflow through each gap is laminar and fully developed. Derive an expression for air mass flow rate ($\dot{m}$) in terms of hot and cold room densities (ρ_h and ρ_c), gap opening and length (D, L), door height and width and height (W, H), ν and g.

(b) Derive an expression for the net convection heat transfer from the warm room to the cold room , through the two gaps in terms of dimensions shown in the figure and fluid properties.

(c) If T_c=10°C, T_h=30°C, D=0.5 mm, L=5 cm, H=2.2 m, and W=1.5 m. Calculate the mass flow rate and convection heat transfer through the two gaps. Comment on how these quantities react to an increase in the gap thickness.

Note: the average fluid velocity for a fully developed laminar flow through a gap of height D, length L is:

$$V_{avg} = \frac{D^2}{12\mu}\frac{\Delta P}{L}$$



LAPLACE TRANSFORM PAIRS

	$f(t)$	$\hat{f}(s)$
1.	1	$\dfrac{1}{s}$
2.	e^{at}	$\dfrac{1}{s-a}$
3.	$\sin at$	$\dfrac{a}{s^2+a^2}$
4.	$\cos at$	$\dfrac{s}{s^2+a^2}$
5.	$\sinh at$	$\dfrac{a}{s^2-a^2}$
6.	$\cosh at$	$\dfrac{s}{s^2-a^2}$
7.	$af(t)$	$a\hat{f}(s)$
8.	$f(t)+g(t)$	$\hat{f}(s)+\hat{g}(s)$
9.	$f'(t)$	$-f(0)+s\hat{f}(s)$
10.	$f''(t)$	$-f'(0)-sf(0)+s^2\hat{f}(s)$
11.	$tf(t)$	$-\dfrac{d}{ds}\hat{f}(s)$
12.	$\displaystyle\int_{\sigma=0}^{t} f(\sigma)g(t-\sigma)\,d\sigma$	$\hat{f}(s)\hat{g}(s)$

	$f(t)$	$\hat{f}(s)$
13.	$\sum_{k=1}^{N} \frac{P(s_k)}{Q'(s_k)} e^{s_k t}$ where the s_k are the roots of $Q(s) = 0$	$\frac{P(s)}{Q(s)}$ where $P(s)$ and $Q(s)$ are polynomials in s
14.	$\mathbf{1}(t-a)$ where $\mathbf{1}(\alpha) = \begin{cases} 0 & \alpha < 0 \\ 1 & \alpha \geq 0 \end{cases}$	$\frac{e^{-as}}{s}$
15.	$g(t-a)\mathbf{1}(t-a)$	$\hat{g}(s)e^{-as}$
16.	$\mathbf{1}(t-a)(1 - e^{-b(t-a)})$	$\frac{be^{-as}}{s(s+b)}$
17.	$t^{\nu/2} J_\nu(a\sqrt{t})$	$\frac{a^\nu}{2^\nu s^{\nu+1}} e^{-a^2/4s}$
18.	$\frac{1}{\sqrt{at}} e^{-bt} I_1(2\sqrt{t/a})$	$e^{1/[a(s+b)]} - 1$
19.	$e^{-bt} I_0(2\sqrt{at})$	$\frac{e^{a/(s+b)}}{s+b}$
20.	$J_0(at)$	$\frac{1}{\sqrt{s^2+a^2}}$
21.	$J_n(at)$	$\frac{a^n}{\sqrt{s^2+a^2}(\sqrt{s^2+a^2}+s)^n}$ where n = nonnegative integer
22.	$\frac{a}{2\sqrt{\pi t^3}} e^{-(a^2/4t)-bt}$	$\exp(-a\sqrt{s+b})$
23.	$\frac{1}{\sqrt{\pi t}} e^{-a^2/4t}$	$\frac{\exp(-a\sqrt{s})}{\sqrt{s}}$
24.	$\operatorname{erfc} \frac{a}{2\sqrt{t}}$	$\frac{\exp(-a\sqrt{s})}{s}$
25.	$2\sqrt{\frac{t}{\pi}} e^{-(a^2/4t)} - a \operatorname{erfc} \frac{a}{2\sqrt{t}}$	$s^{-\frac{3}{2}} \exp(-a\sqrt{s})$
26.	$e^{ab} e^{b^2 t} \operatorname{erfc}\left(b\sqrt{t} + \frac{a}{2\sqrt{t}}\right)$	$\frac{\exp(-a\sqrt{s})}{\sqrt{s}(b+\sqrt{s})}$

	$f(t)$	$\hat{f}(s)$
27.	$\operatorname{erfc}\left(\dfrac{a}{2\sqrt{t}}\right) - e^{b+(b^2t/a^2)}\operatorname{erfc}\left(\dfrac{a}{2\sqrt{t}} + \dfrac{b\sqrt{t}}{a}\right)$	$\dfrac{b\exp(-a\sqrt{s})}{s(b + a\sqrt{s})}$
28.	$-\dfrac{2\pi}{L^2}\sum_{n=1}^{\infty}(-1)^n n \sin\dfrac{n\pi x}{L} e^{-(n^2\pi^2 t)/L^2}$	$\dfrac{\sinh x\sqrt{s}}{\sinh L\sqrt{s}}$ $(-L < x < +L)$
29.	$1 + 2\sum_{n=1}^{\infty}\cos 2n\pi x e^{-n^2\pi^2 t}$	$\dfrac{\cosh[(2x-1)\sqrt{s}]}{\sqrt{s}\sinh\sqrt{s}}$ $(0 \le x \le 1)$

WATER AT ATMOSPHERIC PRESSURE

T (°C)	ρ (g/cm³)	c_P (kJ/kg · K)	c_v (kJ/kg · K)	h_{fg} (kJ/kg)	β (K⁻¹)
0	0.9999	4.217	4.215	2501	-0.6×10^{-4}
5	1	4.202	4.202	2489	$+0.1 \times 10^{-4}$
10	0.9997	4.192	4.187	2477	0.9×10^{-4}
15	0.9991	4.186	4.173	2465	1.5×10^{-4}
20	0.9982	4.182	4.158	2454	2.1×10^{-4}
25	0.9971	4.179	4.138	2442	2.6×10^{-4}
30	0.9957	4.178	4.118	2430	3.0×10^{-4}
35	0.9941	4.178	4.108	2418	3.4×10^{-4}
40	0.9923	4.178	4.088	2406	3.8×10^{-4}
50	0.9881	4.180	4.050	2382	4.5×10^{-4}
60	0.9832	4.184	4.004	2357	5.1×10^{-4}
70	0.9778	4.189	3.959	2333	5.7×10^{-4}
80	0.9718	4.196	3.906	2308	6.2×10^{-4}
90	0.9653	4.205	3.865	2283	6.7×10^{-4}
100[a]	0.9584	4.216	3.816	2257	7.1×10^{-4}

T (°C)	μ (g/cm · s)	ν (cm²/s)	k (W/m · K)	α (cm²/s)	Pr	$\frac{g\beta}{\alpha\nu} = \frac{Ra_H}{H^3\,\Delta T}$ (K⁻¹ · cm⁻³)
0	0.01787	0.01787	0.56	0.00133	13.44	-2.48×10^3
5	0.01514	0.01514	0.57	0.00136	11.13	$+0.47 \times 10^3$
10	0.01304	0.01304	0.58	0.00138	9.45	4.91×10^3
15	0.01137	0.01138	0.59	0.00140	8.13	9.24×10^3
20	0.01002	0.01004	0.59	0.00142	7.07	14.45×10^3
25	0.00891	0.00894	0.60	0.00144	6.21	19.81×10^3
30	0.00798	0.00802	0.61	0.00146	5.49	25.13×10^3
35	0.00720	0.00725	0.62	0.00149	4.87	30.88×10^3
40	0.00654	0.00659	0.63	0.00152	4.34	37.21×10^3
50	0.00548	0.00554	0.64	0.00155	3.57	51.41×10^3
60	0.00467	0.00475	0.65	0.00158	3.01	66.66×10^3
70	0.00405	0.00414	0.66	0.00161	2.57	83.89×10^3
80	0.00355	0.00366	0.67	0.00164	2.23	101.3×10^3
90	0.00316	0.00327	0.67	0.00165	1.98	121.8×10^3
100	0.00283	0.00295	0.68	0.00166	1.78	142.2×10^3

Source: Data collected from Refs. 1–3.

[a]Saturated.

DRY AIR AT ATMOSPHERIC PRESSURE

T (°C)	ρ (kg/m³)	c_P (kJ/kg · K)	μ (kg/s · m)	ν (cm²/s)	k (W/m · K)	α (cm²/s)	Pr	$\frac{g\beta}{\alpha\nu} = \frac{Ra_H}{H^3 \Delta T}$ (cm⁻³ · K⁻¹)
−180	3.72	1.035	6.50×10^{-6}	0.0175	0.0076	0.019	0.92	3.2×10^{4}
−100	2.04	1.010	1.16×10^{-5}	0.057	0.016	0.076	0.75	1.3×10^{3}
−50	1.582	1.006	1.45×10^{-5}	0.092	0.020	0.130	0.72	367
0	1.293	1.006	1.71×10^{-5}	0.132	0.024	0.184	0.72	148
10	1.247	1.006	1.76×10^{-5}	0.141	0.025	0.196	0.72	125
20	1.205	1.006	1.81×10^{-5}	0.150	0.025	0.208	0.72	107
30	1.165	1.006	1.86×10^{-5}	0.160	0.026	0.223	0.72	90.7
60	1.060	1.008	2.00×10^{-5}	0.188	0.028	0.274	0.70	57.1
100	0.946	1.011	2.18×10^{-5}	0.230	0.032	0.328	0.70	34.8
200	0.746	1.025	2.58×10^{-5}	0.346	0.039	0.519	0.68	9.53
300	0.616	1.045	2.95×10^{-5}	0.481	0.045	0.717	0.68	4.96
500	0.456	1.093	3.58×10^{-5}	0.785	0.056	1.140	0.70	1.42
1000	0.277	1.185	4.82×10^{-5}	1.745	0.076	2.424	0.72	0.18

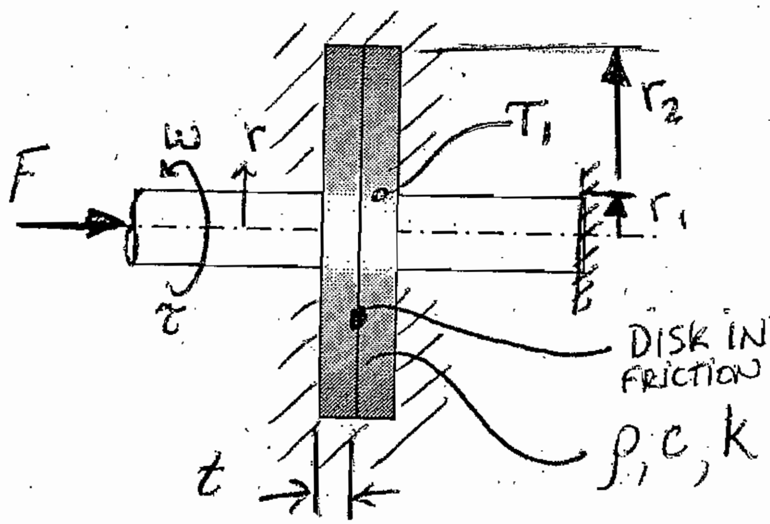
Source: Data collected from Refs. 1–3.

Problem # 1.

In a test to determine the friction coefficient, μ, associated with a disc brake, one disk and its shaft are rotated at a constant angular velocity, ω, while an equivalent disk/shaft assembly is stationary. A known force, F, is applied to the system, and the corresponding torque, τ, required to maintain rotation is measured. The disc contact pressure may be assumed to be uniform and the disks may be assumed to be well insulated from the surroundings.

a) Obtain an expression to evaluate the friction coefficient, μ, from known quantities;

b) For the region $r_1 \le r \le r_2$, determine the radial temperature distribution, $T(r)$, in the disk, where $T(r_1)=T_1$, is given. State your assumptions.



Heat diffusion equation in cylindrical coordinates:

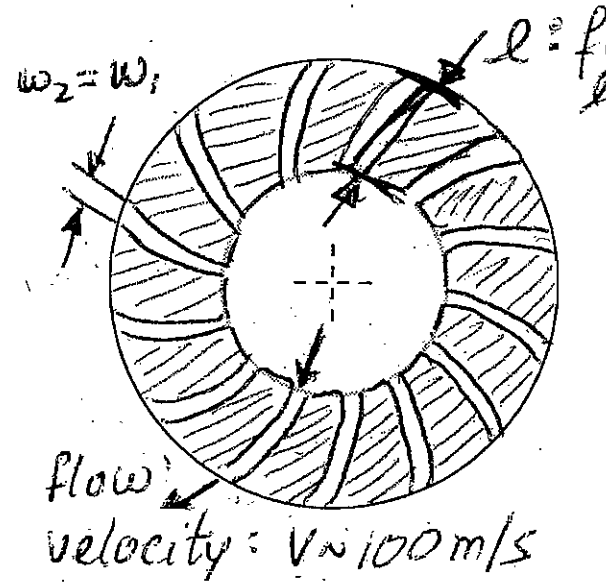
$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}_{gen}=\rho c_p\frac{\partial T}{\partial t}$$

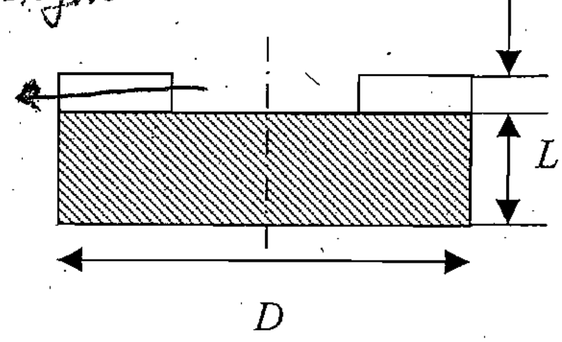
Problem # 2.

We wish to evaluate the impact of scale on the transient thermal behavior of a silicon turbine. Consider a disk with flow passages on one side, forming the turbine as illustrated above. At start-up, the room-temperature rotor is heated by a steady flow of hot gases ($T_\infty = 1200°C$) through the turbine passages. The rotor can otherwise be assumed to be insulated from the surroundings. Two scales are considered: (a) micro-scale: $D = 4$ mm, $L = 0.5$ mm, $w_1 = w_2 = 50$ microns, $l = 1$ mm; $V = 100$ m/s; (b) macro-scale: all dimensions 40 times larger, same flow velocity ($V = 100$ m/s). Channel flow and Nusselt number at the macro scale can be assumed to be the same as at micro scale, for simplicity. Provide first order estimates to the following questions, and clearly state your assumptions and reasoning:

a) Do you expect the temperature to be uniform across the disk thickness as it heats up? What is the effect of scale on the temperature distribution across the disk (i.e. micro vs macro-scale), assuming the same flow characteristics at all scales?

b) What is the steady-state temperature of the rotor and time scale for it to reach steady state (i.e. the transient time response)? What is the effect of scale on the transient time response?





Section view

Useful physical properties and relations:
Hot gases: k = 0.062 W/mK; $\rho = 0.39$ kg/m^3; Pr = 0.7; $\nu = 100 \times 10^{-6}$ m^2/s; $c_p = 1.1$ kJ / kg·K
Silicon: k = 148 W/mK; $\rho = 2300$ kg/m^3; c = 712 J / kg·K

(1 micron = 10^{-6} m)

Problem 3. Consider Couette flow, where the upper plate (located at y=L) moves at U = 10 m/s, and is separated by a distance L = 3 mm from the bottom, stationary plate (located at y = 0). Both plates are infinite in extent with the intervening space filled with engine oil. The top plate is at T_L = 30 °C and the lower plate is at T_o = 10 °C.

(a) The temperature distribution between the plates is given by the equation below. Rearrange the temperature distribution into a dimensionless form, using the following new variables and dimensionless groups:

$$T(y) = T_o + \frac{\mu}{2k}U^2\left[\frac{y}{L} - \left(\frac{y}{L}\right)^2\right] + (T_L - T_o)\frac{y}{L}$$

$\Theta = [T(y)-T_o]/[T_L-T_o]$
$\eta = y/L$ (NOTE: the 'y' coordinate represents the vertical direction)
$Pr = \mu C_p/k$
$E\# = U^2/[C_p(T_L-T_o)]$

(b) Derive an expression that prescribes the conditions under which there will be no heat transfer to the upper plate. (HINT: Make use of the dimensionless groups, i.e. final answer should be in terms of dimensionless groups).

(c) Derive an expression for the heat transfer rate to the lower plate for the conditions identified in part (b).

(d) Sketch a plot of Θ versus η for η between 0 and 1 for values of Pr*E# = 0, 1, 2. Briefly describe the heat transfer rate as Pr*E# increases from 0 to $\geq$ 2.

Relevant Equations-Heat Transfer Problems:

Temperature Distribution for Couette Flow:

$$T(y) = T_o + \frac{\mu}{2\kappa}U^2\left[\frac{y}{L} - \left(\frac{y}{L}\right)^2\right] + (T_L - T_o)\frac{y}{L}$$

$$E = \frac{U^2}{C_p(T_L - T_o)}; \Pr = \frac{\mu C_p}{\kappa}$$

Problem 4. Special coatings are often formed by depositing thin layers of a molten material on a solid substrate. Solidification begins at the substrate surface and proceeds until the thickness S of the solid layer becomes equal to the thickness δ of the deposit. (See Figure below).

(a) Consider conditions for which molten material at its fusion temperature T_f is deposited on a **large** substrate that is at an initial uniform temperature T_i. With $S = 0$ at $t = 0$, develop an expression for estimating the time t_d required to completely solidify the deposit if it remains at T_f throughout the solidification process. Express your result in terms of the substrate thermal conductivity and thermal diffusivity (κ_s, α_s), the density and latent heat of fusion of the deposit (ρ, h_{sf}), the deposit thickness δ, and the relevant temperatures (T_f, T_i). LIST all assumptions and justify. (HINT: Start with an energy balance)

(b) If the substrate were not large, compared to the molten material, briefly suggest an alternate method of solution for part (a)...just a few statements here outlining the proposed alternate method...DO NOT SOLVE.

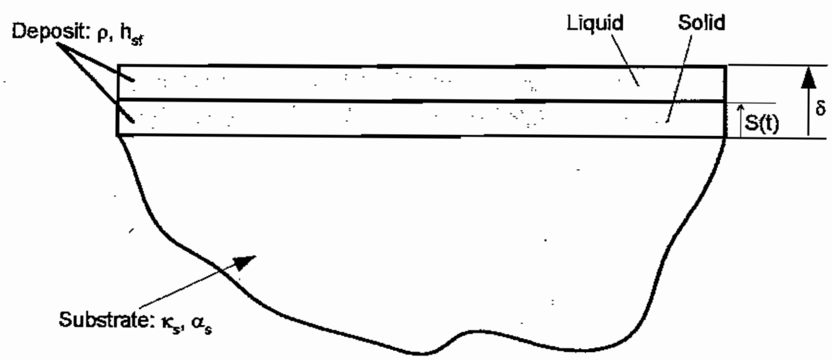


Figure for Problem 2

Relevant Equations-Heat Transfer Problems:

Surface Heat Flux in a Semi-Infinite Solid:

$$q''_{s,conduction} = \frac{\kappa(T_s - T_i)}{\sqrt{\pi \alpha t}}$$

where T_s is the surface temperature and T_i is the initial temperature