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Energy Sources/Conversion **Qualifying Examination** **May 2016**

Ideal Gas Properties at 300K
Ideal Gas behavior: Either $P < 0.1P_c$ OR ($T > 2T_c$ and $P < 5P_c$)

Gas	Mol Wt	R kJ/kgK	c_p kJ/kgK	c_v kJ/kgK	$c_p/c_v = k$
Air	29	0.287	1.00	0.72	1.4
Steam	18	0.46	1.87	1.41	1.33
Nitrogen	28	0.297	1.04	0.745	1.4

SYS= System CV=Control Volume, scs = simple comp subs, sfs=single fluid stream
Processes: REV=Reversible, IRR=Irreversible

Property Relations for any scs: $Tds=du-Pdv$ and $Tds=dh-vdP$

Property Relations for an Ideal Gas: $Pv=RT$ or $PV=mRT$ $dh=c_p dT$ and $du=c_v dT$
$s_2-s_1=c_p \ln(T_2 \backslash T_1) - R \ln(p_2 \backslash p_1)$; $s_2-s_1=c_v \ln(T_2 \backslash T_1) + R \ln(v_2 \backslash v_1)$

Isentropic Relations for an Ideal Gas: $\left(\frac{P_2}{P_1}\right) = \left(\frac{v_1}{v_2}\right)^k$, $\left(\frac{T_2}{T_1}\right) = \left(\frac{v_1}{v_2}\right)^{k-1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$

I Law: SYS: $\delta Q = dE + \delta W$

CV: $\dot{Q}_{cv} + \sum \dot{m}_i (h_i + \frac{1}{2}V_i^2 + gZ_i) = dE_{cv}/dt + \sum \dot{m}_e (h_e + \frac{1}{2}V_e^2 + gZ_e) + \dot{W}_{cv}$

II Law: SYS: $dS = \frac{\delta Q}{T} + \delta S_{gen}$

CV: $dS_{cv}/dt + \sum \dot{m}_e s_e - \sum \dot{m}_i s_i \geq \sum_{cv} \frac{\dot{Q}_{cv}}{T}$

Work for SSSF REV Process: $w = -\int_i^e v dP + \frac{1}{2}(V_i^2 - V_e^2) + g(Z_i - Z_e)$

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Energy Sources/Conversion **Qualifying Examination** **May 2016**

Problem 1

The power conversion in a photovoltaic (PV) cell is primarily dependent on solar irradiance and temperature. At Standard Test Conditions (STC), solar irradiance (E_{rr}) is 1000W/m^2 and temperature (T) is 25°C. The current vs voltage curve (IV curve) for a PV cell is given by the following formula:

$$I = I_{sc} - I_0 \left[e^{\frac{q(V+IR_s)}{\eta kT}} - 1 \right] \qquad [1]$$

V is the cell voltage, I is the cell current, R_s is the cell series resistance, I_0 is the reverse saturation current and I_{sc} is the short circuit current proportional to solar irradiance.
The variable T is temperature in K. In the factor $q/\eta kT$, $q = 1.6$ E-19 C and $k = 1.38$E-23 J/K are constants, η is the module ideality factor.

The photovoltaic modules used in this assignment uses cells with the following characteristics at STC:

$$V_{oc} = 0.6264V \quad I_{sc} = 8.8A \quad R_s = 2.78m\Omega \quad \eta = 1.38$$

V_{oc} is the cell voltage at open circuit ($I = 0$).

At 25°C = 298K, $q/\eta kT$ is 28.19 V^{-1}.

$$V = -I * R_s + 35.47mV \ln\left(1 + \frac{I_{sc} - I}{I_0}\right)$$

Calculate I_0 for this cell at STC and plot the current vs voltage curve for a single cell. Now and plot the current vs voltage curve for 4 modules of 72 cells each, or an equivalent of 288 cells in series.

Problem 2

Consider a single-junction solar photovoltaic cell (see above for description). Using a typical curve as a basis for the cell, show how such a cell would interact with a motor. The torque vs speed equations for a single pole DC motor at steady state are:

$$\omega_m = \frac{V_m - I_m * R_a}{C_e}$$

$$\tau_m = I_m * C_e - \nu_m * \omega_m$$

Where V_m is the voltage applied to the motor, I_m is the current developed at the motor, R_a is the motor resistance, C_e is a motor constant that takes into account materials and design of the motor rotor and stator, ν_m is the motor viscous friction constant. This motor has constant armature self inductance and negligible iron losses. We use a motor with $R_a = 2.8$ ohm, $C_e = 0.27$ V/rad and $\nu_m = 7.96$ E-4 N.m s/rad connected directly with the solar array. Show how you would go about computing the electrical power input and the mechanical power for this motor.

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Problem 3

Air, at a total temperature of 1400 K and a total pressure of 2230 kPa, enters the nozzle row of a turbine stage in the axial direction at the rate of 36.4 kg/s. Other known data are:

Turbine rotor speed	14,000 rpm
Mean blade diameter	48.3 cm
$V_{a3.5}$ (axial velocity at nozzle exit)	170.0 m/s
$\alpha_{3.5}$ (alpha at nozzle exit)	+72.0°
α_4 (alpha at turbine rotor exit)	0.0°

a. Construct the velocity diagrams at the mean blade diameter for this stage
b. Calculate the work and power developed by this stage based on mean diameter conditions
c. Calculate the pressure and temperature at the entry of nozzle (state 3), exit of nozzle (state 3.5, also same as entry to rotor) and exit of the entire stage (state4).

Problem 4

For a 4-stroke Otto cycle based automobile engine:

a.) The following expression for brake power is given:

$$P_b = \eta_i \eta_c \eta_m \eta_v \, \rho_i V_d \frac{N}{X} F Q$$

Assuming that $\eta_i \eta_c$ = 0.30, η_m = 0.85, and η_v = 0.83, F = 1/14.8, ρ_i = 1.2 kg/m^3, Q = 43 MJ/kg, find the brake power for a 4-stroke, 1.6 liter car engine at 6000 rpm. What is the actual brake mean effective pressure under the above conditions?

b.) What does each efficiency term mean? Provide ONE dominant cause of the particular inefficiency and ONE way to improve each of the efficiency term.

Energy Sources/Conversion **Qualifying Examination** **January 2016**

Ideal Gas Properties at 300K
Ideal Gas behavior: Either $P < 0.1P_c$ OR ($T > 2T_c$ and $P < 5P_c$)

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I Law: SYS: $\delta Q = dE + \delta W$

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Work for SSSF REV Process: $w = -\int_i^e v\,dP + \frac{1}{2}(V_i^2 - V_e^2) + g(Z_i - Z_e)$

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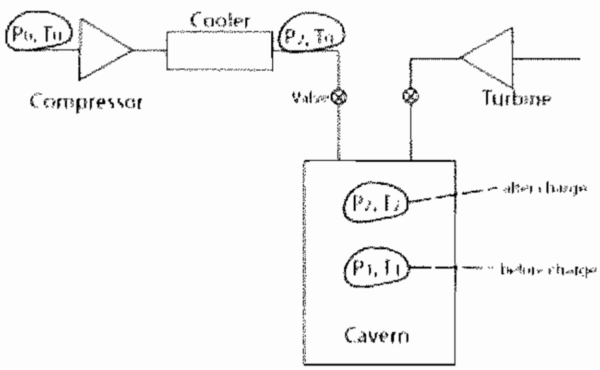
Problem 1 Cheap, large-scale energy storage would do wonders to improve the prospects for integrating intermittent renewable power sources (like wind and solar) into the grid. One currently existing technology is called Compressed Air Energy Storage (CAES). This generally consists of taking a large underground cavern, often a depleted salt cavity, and pressurizing it. When electricity is cheap (e.g., at night), an electric-powered compressor forces more air into the cavern, raising its pressure. When electricity is expensive (e.g., in the afternoon), the air is allowed out of the cavern through a turbine, producing profitable electricity.



In this assignment, we will explore a simple CAES system based on the Huntorf facility, located in North Germany. In our problem, the compressor and turbine will be assumed to be perfectly adiabatic and reversible, so they operate at maximal efficiency. This is unrealistic, but we will see that there are still limits on how well the system can work. In this problem, consider air to be an ideal gas. The cavern of volume V begins with a mass m_1 of air inside it at pressure P_1 and temperature T_1.

Additional air starts at atmospheric pressure and temperature, P_0, T_0. The adiabatic compressor brings the added air to operating pressure P_2 (and a higher temperature). It is then sent through a cooler, where its temperature is brought down to T_0, while maintaining the pressure. This high pressure gas is then forced into the cavern. We add a mass m_i to the cavern, so the final mass of air in the cavern is $m_2=m_1+m_i$. The initial cavern temperature is actually an interesting question. Presumably the first time the cavern is charged, its air begins at $T_1=T_0$. On running the charge/discharge cycle, the cavern is actually cooled down, extracting extra heat from the cavern.

a. Assuming no conduction of heat into the cavern (which is not a good assumption but let us make that assumption anyway), what temperature T_1 will the cavern approach? For the case of the Huntorf CAES plant, with P_1=43 bar, P_2=70 bar, and T_0=298K, what will be the numerical value of T_1? Whether you can or cannot determine T_1 you may leave it as T_1 in your work without substituting this value.

For an ideal gas with constant specific heat, you may a convention where the enthalpy and entropy of air at T_0 and P_0 are zero.

b. Find the work done by the compressor, W_c. Express your answer in terms of m_i, T_0, P_0, P_2. (Hint: use the first law for an adiabatic reversible compression. This expression is only helpful once we know m_i, and we'll reexpress it in terms of the other parameters later in this problem.

c. Consider the control mass of air as it goes through the compressor and the cooler, justify the fact that the work done by the compressor (i.e., $-W_c$) equals the heat lost from the cooler. That is, $Q - W_c = 0$.

d. Applying the first law to the entire system, including cavern, compressor, cooler, and any air that will eventually be in the cavern, show that:

$$Q - W_c = (m_2 u_2 - m_1 u_1) - m_i h_i$$

where u_1 and u_2 are the specific energies of the air in the cavern before and after charging, and hi is the enthalpy of the gas at P_0, T_0. This is the situation of the "transient process".

e. In order to remove m_i from our expressions, we need to find the temperature the cavern comes to after being charged. After the air is pumped into the cavern, the cavern pressure comes to P_2. Determine the cavern temperature T_2:

$$T_2 = T_1 \frac{P_2}{P_1 + \frac{T_1}{T_0}\frac{(P_2 - P_1)}{k}}$$

f. Using this result, find an expression for m_i in terms of P_0, P_1, P_2, T_0, T_1, and V.

g. Use the ideal gas equation of state, re-cast W_c in terms of P_0, P_1, P_2, and V (i.e., remove the dependence on m_i so compressor work can be found). The result does not depend on T1. The result should be:

$$W_c = \frac{V}{k-1}(P_2 - P_1)\left[\left(\frac{P_2}{P_0}\right)^{\frac{k-1}{k}} - 1\right]$$

h. When the air is allowed out of the cavern through the turbine, the pressure of the cavern drops from P_2 to P_1, and the temperature drops from T_2 to some other temperature. Determine the work done on the turbine in bringing the cavern air adiabatically from cavern pressure to P_0.

i. The Huntorf facility has P_1 = 43 bar, P_2 = 70 bar, and a volume of 310,000 m^3. Find the compressor work required to charge the cavern W_c, and the work done by the turbine during expansion W_t.

j. What is the efficiency of the facility, defined as $\eta = \frac{W_t}{|W_c|}$? Recall that the compressor and turbine are assumed to be perfectly efficient. What processes have limited the efficiency of this CAES system?

k. What temperature is the air upon leaving the turbine?

Problem 2

Show the equivalent circuit for a single-junction solar photovoltaic cell showing both the series resistance and the parallel resistances in addition to the diode and the load. Now construct a similar circuit diagram for two of such cells in series. Using this diagram illustrate the effect that shading one of these cells has on the overall performance of the two cell system. Show the resulting effect on a sketch of a V-I curve for the two-cell series arrangement.

Problem 3

Air, at a total temperature of 1400 K and a total pressure of 2230 kPa, enters the nozzle row of a turbine stage in the axial direction at the rate of 36.4 kg/s. Other known data are:

Turbine rotor speed	15,000rpm
Mean blade diameter	48.3 cm
$V_{a3.5}$ (axial velocity at nozzle exit)	170.0 m/s
$\alpha_{3.5}$ (alpha at nozzle exit)	+72.0°
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a. Construct the velocity diagrams at the mean blade diameter for this stage
b. Calculate the work and power developed by this stage based on mean diameter conditions
c. Calculate the pressure and temperature at the entry of nozzle (state 3), exit of nozzle (state 3.5, also same as entry to rotor) and exit of the entire stage (state4).

Problem 4

You are given a 600 kW wind turbine with a diameter of 43 meters. For various bins of wind speed (a bin at 5 m/s represents information from 4.5 to 5.5 m/s) the following information is shown in Table 1, the frequency of wind occurring in a bin, the number of hours in a year that the wind speed occurs, the average turbine wind capacity for that bin and the energy produced by the wind turbine. Assume the density of air to be 1.2 kg/m^3.

a) What is the capacity factor of this wind turbine?
b) What is the Cp value at 7 m/s (ie at the mean wind speed)?

Table 1. Wind Speed Distribution and Wind Turbine Performance

Wind Speed Bin	**Frequency of Occurrence**		**Turbine Capacity**	**Energy Produced**
m/s	%	hr/yr	KW	kWh
4	0.0992	869	7	6000

5	0.1074	941	29	27700
6	0.108	946	63	59500
7	0.1023	896	107	95900
8	0.0919	805	161	129700
9	0.0788	690	224	154600
10	0.0645	565	293	165600
11	0.0507	444	362	160800
12	0.0383	335	423	141800
13	0.0278	243	467	113600
14	0.0194	170	489	83100
15	0.0131	114	497	56800
16	0.0085	74	600	37100
17	0.0053	46	600	23200
18	0.0032	28	600	14000
19	0.0019	16	600	8200
20	0.0011	9	600	4600
21	0.0006	5	600	2500
22	0.0003	3	600	1300
23	0.0002	1	600	700
24	0.0001	1	600	300
25	0	0	600	0
26	0	0	0	0

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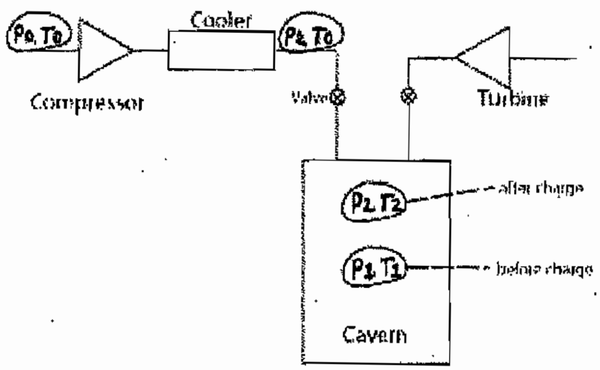
Mechanical Engineering Department, Columbia University
Flex topic: Energy

Problem 1

Cheap, large-scale energy storage would do wonders to improve the prospects for integrating intermittent renewable power sources (like wind and solar) into the grid. One currently existing technology is called Compressed Air Energy Storage (CAES). This generally consists of taking a large underground cavern, often a depleted salt cavity, and pressurizing it. When electricity is cheap (e.g., at night), an electric-powered compressor forces more air into the cavern, raising its pressure. When electricity is expensive (e.g., in the afternoon), the air is allowed out of the cavern through a turbine, producing profitable electricity.



In this assignment, we will explore a simple CAES system based on the Huntorf facility, located in North Germany. In our problem, the compressor and turbine will be assumed to be perfectly adiabatic and reversible, so they operate at maximal efficiency. This is unrealistic, but we will see that there are still limits on how well the system can work. In this problem, consider air to be an ideal gas. The cavern of volume V begins with a mass m_1 of air inside it at pressure P_1 and temperature T_1. Additional air starts at atmospheric pressure and temperature, P_0, T_0. The adiabatic compressor brings the added air to operating pressure P_2 (and a higher temperature). It is then sent through a cooler, where its temperature is brought down to T_0, while maintaining the pressure. This high pressure gas is then forced into the cavern. We add a mass m_i to the cavern, so the final mass of air in the cavern is $m_2=m_1+m_i$.

[The initial cavern temperature is actually an interesting question. Presumably the first time the cavern is charged, its air begins at $T_1=T_0$. On running the charge/discharge cycle, the cavern is actually cooled down, extracting extra heat from the cavern. Assuming no conduction of heat into the cavern (which is not a good assumption), the temperature will approach:

$$T_1 = T_0 \frac{kP_1}{P_2 - P_0}\left[\left(\frac{P_2}{P_1}\right)^{1/k} - 1\right]$$

For the case of the Huntorf CAES plant, with P_1=43 bar, P_2=70 bar, and T_0=298K, this gives T_1=108K, which is very cold! (Note: You should assume T_1 as written above for the purposes of this problem, but you may leave it as T_1 in your work without substituting in this expression.)

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Mechanical Engineering Department, Columbia University
Flex topic: Energy

For an ideal gas with constant specific heat, the following equations may be helpful. These include a convention where the enthalpy and entropy of air at T_0 and P_0 are zero. They assume different origins for the internal energy, enthalpy, and entropy scales and so their use will lead to inconsistencies that will give wrong answers. As usual, lower case quantities (h, u, s) are specific or per kg values.

$$m = \frac{PV}{RT} \leftarrow \text{Ideal Gas Law}$$

$$h = C_p(T - T_0) \leftarrow \text{Nonstandard definition of zero of enthalpy, implying:}$$

$$u = h - RT = C_v T - C_p T_0$$

$$R = C_p - C_v$$

$$k = \frac{C_p}{C_v}$$

$$s = C_p \ln\left(\frac{T}{T_0}\right) - R\ln\left(\frac{P}{P_0}\right)$$

$$\frac{T_a}{T_b} = \left(\frac{P_a}{P_b}\right)^{\frac{k-1}{k}} \leftarrow \text{for ideal gases in adiabatic processes}$$

a) Find the work done by the compressor, W_c. Express your answer in terms of m_i, T_0, P_0, P_2. (Hint: use the first law for an adiabatic reversible compression. This expression is only helpful once we know mi, and we'll reexpress it in terms of the other parameters later in this assignment.)

b) Applying the first law of thermodynamics to a control mass of air as it goes through the compressor and the cooler, justify the statement that the work done by the compressor (i.e., $-W_c$) equals the heat lost from the cooler. That is, $Q - W_c = 0$.

Applying the first law to the entire system, including cavern, compressor, cooler, and any air that will eventually be in the cavern, it can be shown that:

$$Q - W_c = (m_2 u_2 - m_1 u_1) - m_i h_i$$

where u_1 and u_2 are the specific energies of the air in the cavern before and after charging, and hi is the enthalpy of the gas at P_0, T_0. This is the situation of the "transient process". You do not need to derive this expression.

In order to remove m_i from our expressions, we need to find the temperature the cavern comes to after being charged. After the air is pumped into the cavern, the cavern pressure comes to P_2.

c) Show that the cavern temperature T_2 becomes:

$$T_2 = T_1 \frac{P_2}{P_1 + \frac{T_1}{T_0}\frac{(P_2 - P_1)}{k}}$$

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d) Using this result, find an expression for m_i in terms of P_0, P_1, P_2, T_0, T_1, and V.

At this point we basically know what is needed to calculate the overall efficiency of the CAES system, but we now consider what exactly is being stored in the cavern, given your answer to above.

Use the ideal gas equation of state, re-cast W_c in terms of P_0, P_1, P_2, and V (i.e., remove the dependence on m_i so compressor work can be found). The result does not depend on T1.

e) The result should be:

$$W_c = \frac{V}{k-1}(P_2 - P_1)\left[\left(\frac{P_2}{P_0}\right)^{\frac{k-1}{k}} - 1\right]$$

When the air is allowed out of the cavern through the turbine, the pressure of the cavern drops from P_2 to P_1, and the temperature drops from T_2 to some other temperature.

f) Show that the work done on the turbine in bringing the cavern air adiabatically from cavern pressure to P_0 is:

$$W_t = \frac{V}{k-1}(P_2 - P_1)\left[1 - \left(\frac{P_0}{P_1}\right)^{\frac{(k-1)}{k}}\right]$$

The Huntorf facility has P_1 = 43 bar, P_2 = 70 bar, and a volume of 310,000 m^3. Find the compressor work required to charge the cavern W_c, and the work done by the turbine during expansion W_t.

g) What is the efficiency of the facility, defined as $\eta = \frac{W_t}{|W_c|}$? Recall that the compressor and turbine are assumed to be perfectly efficient. What processes have limited the efficiency of this CAES system? What temperature is the air upon leaving the turbine?

Problem 2

Air at 100 kPa and 300 K enters an axial-flow compressor stage with a velocity of 200 m/s. The stage has a tip diameter of 60 cm, a hub diameter of 50 cm, and rotates at 10,000 rpm. The air enters and leaves the stage axially, with no change in axial velocity or mean radius. The air is turned through 18° as it passes through the rotor blades. Assuming constant specific heats with k = 1.4 and assuming that the air enters and leaves the blades at the blade angles (i.e. the flow relative to the blade is aligned with the blade):

a. Construct the velocity diagrams at the mean blade height of this stage. Show blade shape schematic (being especially careful about the blade angles at the leading and trailing edges). Show all relevant absolute and relative velocities and blade angles for both the rotor and the stator stage. Clarity of the sketches (make them reasonable large so that one can easily see your work) and labeling of all quantities is an essential element of responding to this question.
b. Calculate the mass flow rate

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c. Calculate the power required to drive this compressor stage.

Problem 3

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19	0.0019	16	600	8200
20	0.0011	9	600	4600
21	0.0006	5	600	2500
22	0.0003	3	600	1300
23	0.0002	1	600	700
24	0.0001	1	600	300
25	0	0	600	0
26	0	0	0	0